INVITED TALKS

Quantum chaos of generic systems: Divided phase space, localization of chaotic eigenstates and spectral statistics

Marko Robnik (CAMTP, University of Maribor)
Quantum chaos (or wave chaos) is the study of phenomena in the quantum domain (or in other wave systems) which correspond to the classical chaos. Generic Hamiltonian systems are those with divided classical phase space: regular motion on invariant tori for some initial conditions, and chaotic motion for complementary initial conditions. Quantally, in the semiclassical limit, we have to separate regular and chaotic eigenstates, which we do not do only conceptually, but also mathematically. Furthermore, we study the structure of chaotic eigenstates. If the quantal Heisenberg time (Planck constant divided by the average energy spacing of discrete energy spectra) is shorter than the relevant classical transport time (diffusion time, or relaxation time), we find localized chaotic eigenstates, and extended chaotic eigenstates otherwise. I shall quantify these ideas, and present also the manifestation of these phenomena in the statistical properties of the discrete energy spectra.

Dynamical Glass

Sergej Flach (Center for Theoretical Physics of Complex Systems, Institute for Basic Science, Daejeon, Korea)
Classical many body interacting systems are typically chaotic and their microcanonical dynamics ensures that time averages and phase space averages are identical. In proximity to an integrable limit the properties of the network of nonintegrable action space perturbations help decide whether ergodicity will hold arbitrarily close to the limit (albeit with diverging relaxation times), or whether the system fragments into regular and chaotic parts and enters a nonergodic dynamical glass phase at a finite distance to the integrable limit. This dynamical glass phase is induced by coherent localized excitations - generalized discrete breathers - with diverging averages of lifetimes.


Barred galaxies as examples of demonstration for the role of normally hyperbolic invariant manifolds in dynamical systems with 3 degrees of freedom

Christof Jung (ICF-UNAM, Mexico)
The most important invariant subsets in the phase space are normally hyperbolic invariant surfaces (NHIMs) of codimension 2. For more than 2 degrees of freedom (dof) they play the same role as hyperbolic periodic orbits in the flow or hyperbolic fixed points in the Poincaré map.
play for 2-dof systems. Namely they and their stable and unstable manifolds direct the global
dynamics to a rather large extent. Usually we find NHIMs of codimension 2 over index-1 saddles
of the effective potential. A barred galaxy is a nice 3-dof example of demonstration for the
importance and for the typical properties of these objects. In this particular example we even
have a surprising relation between the unstable manifolds of the NHIMs and directly observable
structures in the position space.

Exponentially fast dynamics in the Fock space of chaotic many-body systems
Lea F. Santos (Yeshiva University)
We demonstrate analytically and numerically that in isolated quantum systems of many
interacting particles, the number of states participating in the evolution after a quench increases
exponentially in time, provided the eigenstates are delocalized in the energy shell. The rate of the
exponential growth is defined by the width $\Gamma$ of the local density of states (LDOS) and
is associated with the Kolmogorov-Sinai entropy for systems with a well defined classical limit.
In a finite system, the exponential growth eventually saturates due to the finite volume of the
energy shell. We estimate the time scale for the saturation and show that it is much larger than
the characteristic decay time of the initial state $1/\Gamma$. Numerical data obtained for a two-
body random interaction model of bosons and for a dynamical model of interacting spin-1/2
particles show excellent agreement with the analytical predictions.

Chaos-induced spin topological structure in kicked rotor
Chushun Tian (CAS Key Laboratory of Frontiers in Theoretical Physics and Institute of
Theoretical Physics, Chinese Academy of Sciences, Beijing)
The kicked rotor is a “standard model” in studies of nonlinear dynamics. The kicked rotor
without spin degree of freedom nowadays has been well studied. In this talk, I will present our
recent result for a kicked rotor with spin degree of freedom. We find a dynamical phenomenon
mathematically equivalent to the integer quantum Hall effect occurs, where Planck’s quantum
mimics the magnetic field. I will show that this phenomenon is of chaos origin.

Quantum Lyapunov Exponents
Victor Galitski (Joint Quantum Institute, University of Maryland, U.S.A.)
Classical chaotic systems exhibit exponential divergence of initially infinitesimally close
trajectories, which is characterized by the Lyapunov exponent. This sensitivity to initial
conditions is popularly known as the “butterfly effect.” Of great recent interest has been to
understand how/if the butterfly effect and Lyapunov exponents generalize to quantum
mechanics, where the notion of a trajectory does not exist. In this talk, I will introduce the
measure of quantum chaoticity − so called out-of-time-ordered four-point correlator (whose
semiclassical limit reproduces classical Lyapunov growth), and use it to describe quantum
chaotic dynamics and its eventual disappearance in the standard models of classical and quantum
chaos − Bunimovich billiard and standard map or kicked rotor. I will also mention our recent
results on quantum Lyapunov exponent in interacting disordered metals, which exhibit an
interaction-induced transition from quantum chaotic to non-chaotic dynamics, which may
manifest itself in a sharp change of the distribution of energy levels from Wigner-Dyson to Poisson statistics.

*Localization of light in subradiant Dicke states: a mobility edge in the imaginary axis*

**Giuseppe Luca Celardo** (Institute of Physics, IFUAP-BUAP, Puebla, Mexico)

Anderson localization of light in three dimensions has challenged experimental and theoretical research for the last decades. Localization of light in cold atomic systems presents strong differences from the standard problem of localization since one needs to deal with an open quantum wave problem in presence of long range hopping which induces strong cooperative effects, such as super and subradiance. Contrary to common believe, we show that localization of light is possible in the dilute regime for subradiant states. Additional disorder in atomic transition frequencies leads to the emergence of a mobility edge in the immaginairy axis, independent of the real energy. The existence of a critical lifetime above which subradiant Dicke states are localized appears as a general feature of scalar wave localization. A preliminary analysis also indicates that the localization length diverges as a power law at such critical lifetime.


*Diluted banded random matrices: scaling behavior of eigenfunction and spectral properties*

**Jose Antonio Mendez-Bermudez** (IFUAP, Puebla)

We demonstrate that the normalized localization length $\beta$ of the eigenfunctions of diluted (sparse) banded random matrices follows the scaling law $\beta=\frac{x}{1+x}$. The scaling parameter of the model is defined as $x \propto (b_{\text{eff}}^2/N)^{\delta}$, where $b_{\text{eff}}$ is the average number of non-zero elements per matrix row, $N$ is the matrix size, and $\delta \sim 1$. Additionally, we show that $x$ also scales the spectral properties of the model (up to certain sparsity) characterized by the spacing distribution of eigenvalues. Our results may have direct application to multilayer random networks whose adjacency matrices are diluted banded random matrices.

*Quantum Chaos, Information Scrambling and Intrinsic Decoherence in Many Body Systems: An Experimental and Theoretical Approach Through the Loschmidt Echo*

**Horacio M. Pastawski** (Instituto de Física Enrique Gaviola (CONICET-UNC) and Facultad de Matemática, Astronomía, Física y Computación (FaMAF), Universidad Nacional de Córdoba, 5000, Córdoba, Argentina)

Our “Central Hypothesis of Irreversibility” states that many-body dynamics at high temperatures is chaotic and hence should present a form of “Butterfly effect” or Lyapunov instability towards time-reversal in a Loschmidt Echo experiment. [1] The irreversibility time-scale $T_3$ should be the tied to the many-body mixing time $T_2$ fixed by the second moment of the dipolar Hamiltonian. To test this, we implement a pulse sequence which allows the system to evolve with scaled dipolar Hamiltonian in polycrystalline Adamantane. The pulse sequence is a mixture
of the Proportionally Refocused Loschmidt echo (PRL echo) [3] and the standard Magic echo (ME). A Forward and/or a backward dynamics are characterized by a zero order Hamiltonian representing the homonuclear dipolar interaction $H^o$ modulated by a factor $k$. The experiments were carried out with different scaling factors to analyze the evolution of the total magnetization and the Loschmidt echo. In all cases, a strong dependence between the evolution rate and the scaling factor is observed. Remarkably, all the curves are not exponential but Gaussian and, except those with very small $k$, scale into a single one when plotted against $T_2 = \frac{T^o_2}{k}$, the characteristic time of the scaled dipolar Hamiltonian. This occurs in spite of the fact that experimental errors, the ultimate source of irreversibility, do not scale with $T_2$. In other words, irreversibility time scales only with the mixing time of the forward dynamics, i.e. $T_3 \approx 3.55 T_2$ for all scaling factors. For too slow dynamics [4,5], however, experimental errors take over and dominate irreversibility.


Failure of thermalisation in small equilibrium systems

F. Leyvraz, Hernán Larralde and Alberto Salazar (ICF-UNAM)

We consider the statistical mechanics of a small system subject to a constant external field. As is well known, in the canonical ensemble the system i) obeys a barometric formula for the density profile and ii) the kinetic temperature is independent of height, even when the system is small. We show here that in the microcanonical ensemble the kinetic temperature of the particles affected by the field is not constant with height, but that rather, generally speaking, it decreases with a gradient of order $1/N$. Even more, if we have a mixture of two species, one which is influenced by the field and the other which is not, we find that the two species' kinetic temperatures are generally different, even at the same height. These facts are shown in detail by studying a simple mechanical model: a Lorentz Gas where particles and spinning disks interact and the particles are subjected to a constant external force. In the microcanonical ensemble, the kinetic temperature of the particles is indeed found to vary with height; the disks’ kinetic temperature, on the other hand, is height-independent, and thus, differs from that of the particles with which they interact.

On the location of Saturn's F ring

Luis Benet (ICF-UNAM)
Impact of Thermal Effects on the Orbital Evolution of Low-mass Protoplanets

Frederic Masset (ICF-UNAM)

I will briefly review the basics of planet-disc tidal interactions, both for the low (terrestrial) mass regime and high mass regime, relevant for gaseous giant planets. I will then show how a finite thermal diffusion in the disc, and the release of heat into the ambient disc by a hot, accreting low-mass planet can have a strong impact on the time evolution of its orbital elements. One of the main conclusions is that sufficiently luminous planetary embryos may experience a growth of eccentricity and inclination to values comparable to the disc's aspect ratio, in sharp contrast with the conventional wisdom that the gaseous disc damps the eccentricity and inclination.
CONTRIBUTED TALKS

Chaotic quantum states

Jorge G. Hirsch (ICN-UNAM)

Quantitative quantum measures of chaos, quantum equivalents of the Lyapunov exponent, are investigated in the simplest non-integrable atom-field system, the Dicke model. The semiclassical dynamics is described with the expectation value of the Hamiltonian between coherent states, allowing for a complete characterization of classical regularity and chaos in terms of the coupling strengths and the energy of the classical system [1]. Employing efficient diagonalization techniques [2] we have also performed a detailed quantitative study of the quantum dynamics in regular and chaotic regions [3]. The ratio of growth of the Participation Ratio (PR) of the coherent states, expanded in the basis of Hamiltonian eigenstates, as the number of atoms increases, is a clear signal of quantum chaos [4]. The Out of Time Ordered Correlator (OTOC) has an exponential increase at early times, before saturation. These exponents can be evaluated for any quantum state in the atom-photon system, and are a clear signature of a chaotic quantum behavior. Their close correlation with the Lyapunov exponents is exhibited [5].


Measurement of mode-to-mode transmission in chaotic wave scattering

Rafael Méndez (ICF, UNAM)

Using two methods, measurements of the partial transmissions of out-of-plane waves through a two-dimensional elastic chaotic billiard, are reported. In the first method electromagnetic-acoustic transducers designed ad-hoc selectively excite and detect mode-to-mode transmissions. In the second method the partial transmissions the mode-to-mode transmissions are obtained from point-to-point measurements. The obatined distributions are compared with those comming from random matrices showing a good agreement for systems with/without time reversal symmetry. From the partial transmissions the total transmission, or dimensionless mechanical conductance, is constructed and its distribution agrees with the predictions of random matrix-theory. A very striking result is obtained: the mode-to-mode distributions show a peak for $T=0$ while the total conductance has a peak at $ST=1$.

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Quantum chaos from the dynamics of initial coherent states in a two-degrees of freedom Hamiltonian

Sergio Lerma-Hernández

The dynamics of initial Coherent states is investigated in a simple non-integrable model, the Dicke model, which describes the coupling between a set of N two level systems and a bosonic mode. For large enough couplings, the classical model exhibits a transition from regular to chaotic dynamics as a function of the excitation energy. We compare the classical regular-chaos transition with quantum results for initial coherent states, specifically with the number of Hamiltonian eigenstates participating in a given initial state and its survival probability as a function of time.

Intermittent Dynamics at Equilibrium and Weak Non-Ergodicity in Many-Body Systems

C. Danieli (Center for Theoretical Physics of Complex Systems, Institute for Basic Sciences, Daejeon, Korea)

The equilibrium value of an observable defines a manifold in the phase space of an ergodic and equipartitioned many-body system. A typical trajectory pierces that manifold infinitely often as time goes to infinity. We use these piercings to measure the fluctuations of the dynamics in equilibrium of the system. We found that close to integrability the equipartitioned dynamics is characterized by a power-law distribution of excursion times far off equilibrium. Long excursions arise from sticky dynamics close to coherent localized time-periodic solutions present in the phase space. Measuring the exponent allows to predict the transition into non-ergodic dynamics. In this contribution, we discuss our findings in the framework of the Fermi-Pasta-Ulam (FPU) and the Klein-Gordon (KG) lattices. There, we found that in the limit of small and large energies respectively, the systems show signatures of ergodicity breaking due to q-breathers (FPU) and discrete breathers (KG).

Transport studies in three-terminal microwave graphs with orthogonal, unitary, and symplectic symmetry

Angel Martinez Argüello (IFUAP, Puebla)

The Landauer-Büttiker formalism establishes an equivalence between the electrical conduction through a device, e.g. a quantum dot, and the transmission. Guided by this analogy we perform transmission measurements through three-port microwave graphs with orthogonal, unitary, and symplectic symmetry thus mimicking three-terminal voltage drop devices. One of the ports is placed as input and a second one as output, while a third port is used as a probe. Analytical predictions show good agreement with the measurements in the presence of orthogonal and unitary symmetries, provided that the absorption and the influence of the coupling port are taken into account. The symplectic symmetry is realized in specifically designed graphs mimicking spin 1/2 systems. Again a good agreement between experiment and theory is found. For the symplectic case the results are marginally sensitive to absorption and coupling strength of the port, in contrast to the orthogonal and unitary case.
Quenched many-body quantum dynamics using $q$-Hermite polynomials

Manan Vyas (ICF, UNAM)

In a $m$ particle quantum system, one can have $k=1,2,\ldots,m$ body interactions. The rank of interactions and the nature of particles (fermions or bosons) can strongly affect the dynamics of the system. To explore this in detail, we study quenched quantum dynamics in many particle systems varying rank of interactions, both for fermionic and bosonic particles. We represent the system Hamiltonian by Fermionic Embedded Gaussian Orthogonal Ensembles (FEGOE) and Bosonic Embedded Gaussian Orthogonal Ensembles (BEGOE) respectively. We show that generating function for $q$-Hermite polynomials describes the semi-circle to Gaussian transition in spectral densities of FEGOE$(k)$ and BEGOE$(k)$ (also the Unitary variants FEGUE$(k)$ and BEGUE$(k)$) as a function of rank of interactions $k$. Importantly, numerical Fourier transform of generating function of $q$-Hermite polynomials explains the short-time decay of survival probability in FEGOE$(1+k)$ and BEGOE$(1+k)$. The parameter $q$ describing these properties is related to excess parameter $\gamma_2$ and we give a formula for FEGOE$(k)$, FEGUE$(k)$, BEGOE$(k)$ and BEGUE$(k)$. We illustrate that the dynamics strongly depends on the rank of interactions and nature of particles and these universal features may be relevant to modeling non-equilibrium quantum systems.

Signatures of chaos and thermalization in the dynamics of many-body quantum systems

Eduardo Jonathan Torres-Herrera (IFUAP, Puebla)

Chaos is now broadly accepted as a crucial ingredient for the onset of thermalization in isolated many-body systems. Most efforts to reveal chaotic properties of quantum systems are based on the analysis of static properties. A theoretically and experimentally useful alternative is to study the dynamics. We present a generic picture of the time evolution of isolated lattice many-body quantum systems. We argue that when the initial state is chaotic, the decay of the survival probability is described by a power-law behavior with exponent caused by the bounds in the spectrum, and thus anticipating thermalization. At times beyond the power-law decay, level repulsion, a typical fingerprint of chaotic systems, manifests itself as a dip below the saturation value of the survival probability, known as the correlation hole.

Finite time singularities at the surface of two-dimensional broken parity fluids

S. Ganeshan (City College of NY CUNY, USA)

In everyday fluids, the viscosity is the measure of resistance to the fluid flow and has a dissipative character. Avron, Seiler, and Zograf showed that viscosity of a quantum Hall (QH) fluid at zero temperature is non-dissipative. This non-dissipative viscosity (also known as ‘odd’ or ‘Hall’ viscosity) is the antisymmetric component of the total viscosity tensor and can be non-zero for parity violating fluids. I will discuss free surface dynamics of a two-dimensional incompressible fluid with the odd viscosity. For the case of incompressible fluids, the odd viscosity manifests itself through the free surface (no stress) boundary conditions. We first find the free surface wave solutions of hydrodynamics in the linear approximation and study the dispersion of such waves. As expected, the surface waves are chiral. In the limit of vanishing shear viscosity and gravity, we derive effective nonlinear Hamiltonian equations for the surface dynamics, generalizing the linear solutions to the weakly nonlinear case. In a small surface
angle approximation, the equation of motion results in a new class of non-linear chiral dynamics which we dub as \textit{chiral Burgers} equation. For generic multiple pole initial conditions, the system evolves to the formation of singularities in a finite time similar to the case of an ideal fluid without odd viscosity.
Scattering and dynamics in one-dimensional PT-symmetric tight-binding models

L. Moreno Rodriguez (Instituto de Física, BUAP, Puebla)

We study two setups of a one-dimensional tight-binding model with balanced gain/loss diagonal terms. In the first model with perfect leads attached to the scattering region, the attention is payed to the unidirectional reflectivity that is due to exceptional points emerging in the energy spectra. Our main findings are the new effects that appear when couplings between neighboring sites are non-symmetric. In the second model with closed boundaries we study the dynamics of wave packets when it is strongly influenced by exceptional points. Our predictions can be be used in experiments with the wave propagation along one-mode waveguides with asymmetric next-neighbor couplings.

Quench dynamics in the Fock space for randomly interacting Fermi particles

Juan Carlos Molina Victoria (IFUAP, Puebla)

We demonstrate analytically and numerically that in isolated quantum systems of many interacting particles, the number of states participating in the evolution after a quench increases exponentially in time provided the eigenstates are delocalized in the energy shell. The rate of the exponential growth is defined by the width $\Gamma$ of the local density of states (LDOS) and is associated with the Kolmogorov-Sinai entropy for systems with a well-defined classical limit. In a finite system, the exponential growth eventually saturates due to the finite volume of the energy shell. We estimate the time scale for the saturation and show that it is estimated as $N/\Gamma$ where $N$ is the number of particles. Numerical data obtained for a two-body random interaction model of fermions fully correspond to the semi-analytical expressions.

Route of chaos in Atom-Photon systems

Jorge Chávez Carlos (UNAM)

The route to chaos in an atom-photon system described by the Dicke Hamiltonian is investigated in the classical and quantum regimes. From the classical Hamiltonian, built employing coherent states, the classical trajectories are obtained for a selected set of initial conditions. Bifurcation in their frequencies mark the surge of chaos, quantified with their Lyapunov exponents. In a numerical tour de force, the participation ratio $\text{PR}$ of the coherent states in the eigenstate basis, along the same paths in phase space, are calculated for systems with up to 200 atoms. A close correspondence is found between the scaling of the participation ratio with the number of atoms and the Lyapunov exponent.

Ergodic to Nonergodic transition in DNLS Lattice

Yagmur Kati, Mithun Thudiyangal, Carlo Danieli, Sergej Flach (Center for Theoretical Physics of Complex Systems, Institute for Basic Science, Daejeon, Korea)

The microcanonical Gross--Pitaevskii (aka semiclassical Bose-Hubbard) lattice model dynamics is characterized by a pair of energy and norm densities. The grand canonical Gibbs distribution fails to describe a part of the density space, due to the boundedness of its kinetic energy spectrum. We define Poincare equilibrium manifolds and compute the statistics of
microcanonical excursion times off them. The tails of the distribution functions quantify the proximity of the many-body dynamics to a weakly-nonergodic phase, which occurs when the average excursion time is infinite. We find that a crossover to weakly-nonergodic dynamics takes place inside the non-Gibbs phase, being unnoticed by the largest Lyapunov exponent. In the ergodic part of the non-Gibbs phase, the Gibbs distribution should be replaced by an unknown modified one. We relate our findings to the corresponding integrable limit, close to which the actions are interacting through a short range coupling network.

**Universal Level Statistics of the Out-of-Time-Ordered Operator**

**Efim B. Rozenbaum, Sriram Ganeshan, and Victor Galitski** (University of Maryland, College Park)

The out-of-time-ordered correlator (OTOC) has been proposed as an indicator of chaos in quantum systems due to its simple interpretation in the semiclassical limit. In particular, its rate of possible exponential growth at $\hbar \to 0$ is closely related to the classical Lyapunov exponent. We explore how this approach to ``quantum chaos'' relates to the random-matrix theoretical description. To do so, we introduce and study the level statistics of the logarithm of the out-of-time-ordered operator, $\hat{\Lambda}(t) = \ln \left( - \left[ \hat{x}(t), \hat{p}(0) \right] \right)/(2t)$, that we dub the "Lyapunovian" or "Lyapunov operator" for brevity. The Lyapunovian's level statistics is calculated explicitly for the quantum stadium billiard. It is shown that in the bulk of the filtered spectrum, this statistics perfectly aligns with the Wigner-Dyson distribution. Our results show that the Lyapunov operator may serve as a useful tool to characterize ``quantum chaos'' and in particular quantum-to-classical correspondence in chaotic systems, by connecting the semiclassical Lyapunov growth that we demonstrate at early times, when the quantum effects are weak, to universal level repulsion that hinges on strong quantum interference effects.

**Temperature of a single chaotic eigenstate**

**Francesco Mattiotti** (Università Cattolica del Sacro Cuore)

The onset of thermalization in a closed system of randomly interacting bosons at the level of a single eigenstate is discussed. We focus on the emergence of Bose-Einstein distribution of single-particle occupation numbers, and we give a local criterion for thermalization dependent on the eigenstate energy. We show how to define the temperature of an eigenstate, provided that it has a chaotic structure in the basis defined by the single-particle states. The analytical expression for the eigenstate temperature as a function of both interparticle interaction and energy is complemented by numerical data.


**Real and imaginary energy gaps: a comparison between single excitation Superradiance and Superconductivity**

**Nahum Calderon Chavez** (Institute of Physics, IFUAP-BUAP, Puebla, Mexico)

A comparison between the single particle spectrum of the discrete Bardeen-Cooper-Schrieffer (BCS) model, used for small superconducting grains, and the spectrum of a paradigmatic model
of Single Excitation Superradiance is presented. Specifically, we study analytically the conditions under which a gapped state emerges in an equally spaced energy spectrum (Picket Fence) due to two different all-to-all couplings: a real and an imaginary one. While the former corresponds to the discrete BCS-model describing the coupling of Cooper pairs in momentum space and it induces a Superconductive regime, the latter describes the coupling of single particle energy levels to a common decay channel and it induces a Superradiant transition. We show that the transition to a Superradiant regime can be connected to the emergence of an imaginary energy gap, similarly to the transition to a Superconductive regime where a real energy gap emerges. The critical coupling at which the Superradiant gap appears is found to be independent of the system size $N$, in contrast with the critical coupling at which the Superconductivity gap appears, which scales as $(\ln N)^{-1}$. The Superradiant and the Superconducting gaps are shown to have the same magnitude in the large gap limit. Robustness to perturbations is shown to occur even in presence of a gap in the imaginary energy axis.


Scaling properties of random networks with losses and gain
Claudia Teresa Martínez Martínez (IFUAP, Puebla)
In this work we study spectral properties of a random network model based on Erdös-Renyi graphs including losses and gain. This model is characterized by three parameters: The network size $N$, the network connectivity $\alpha$, and the loss and gain magnitude $\gamma$. By the use of numerical simulations we propose a scaling parameter $\xi \equiv \xi(N, \alpha, \gamma)$ that fixes the spectral properties of our random network model. This model can be considered as a reference model for realistic networks with losses and gain such as power networks, electronic networks, etc.

Boundary conditions of resonant states and one level models
W. Rodríguez-Cruz, G. Luna-Acosta and A.A. Fernández-Marín (IF, UNAM)
We are interested in obtaining analytical expressions for various scattering functions and for the poles of the S-matrix to gain a deeper understanding between features of scattering functions and spectral properties of a given system. For this purpose we use Wigner’s Reaction Matrix Theory and look for the appropriate one-level approximation for the reaction matrix $R_B = \frac{1}{\beta} (\Phi_{\{B,n\}} | k\rangle \langle k| - k^2 \Phi_{\{B,n\}}) + \beta_{\{B,n\}}$. The first term is the one-level approximation and the second one is the “background” at a given value of the wave number $k$. $B$ is the boundary value parameter defined by the logarithmic derivative of the Reaction Matrix basis functions $\Phi_{\{B,n\}}(x)$ evaluated at the interaction radius $x = a$. $B$ is chosen to be equal to the logarithmic derivative of the actual scattering wave function $\psi(x; k)$ at $x = a$ and at resonance $k = k^*$. It is shown that the one-level approximation works very well as long as $0 \leq B < 1/a$; the smaller $B$, the better. For $B$ larger than some critical value, “underground states” are needed to complete the basis and hence it must be added to the one level model. We show analytically that the effect of the background $\beta$ increases with the magnitude of $B$. 
Cooling limit of a quantum thermocouple
Dr. Charles Stafford*, Dr. Abhay Shastry*, Marco Antonio Jimenez Valencia** (University of Arizona*, Universidad de Sonora**) 

A quantum thermocouple is investigated beyond linear response using the method of nonequilibrium Green's functions (NEGF). Insights on the maximum cooling power and coldest temperature achievable are calculated through an effective field theory of the interacting pi-electrons which characterize molecular junctions' transport properties. The limits on cooling are determined by the interplay of the Peltier effect and Joule heating. Different junction couplings (such as sequential exchange and super-exchange) in a quantum thermocouple are tested.