# QUANTUM OPEN SYSTEMS, DECOHERENCE AND QUANTUM PROCESS TOMOGRAPHY

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APLICATIONS OF QUANTUM MECHANICS
CUERNAVACA, MEXICO
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# PLAN

**Lecture 1: Decoherence and the quantum origin of the classical world. Evolution of quantum open systems. Quantum Brownian motion as a paradigm. Master equation.**

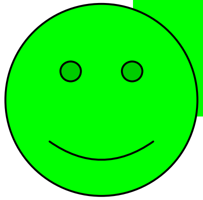
**Lecture 2: General results on Dynamics and Thermodynamics of linear quantum open systems. Emergence of the laws of thermodynamics. Decoherence in Quantum Brownian Motion.**

**Lecture 3: Decoherence timescales. Pointer states. Decoherence and disentanglement. Different dynamical phases for the behavior of quantum correlations in quantum open systems.**

**Lecture 4: Decoherence in quantum information processing. How to fight against decoherence? How to characterize decoherence? Quantum process tomography (QPT).**

**Lecture 5: New methods for Quantum Process Tomography. "Selective and efficient QPT". Theory and experimental implementation with single photons.**

# Lecture 1: The Good Side of Decoherence



Helps us to understand the origin of classical world

Enemy for quantum information processing

*Colaborations with: W. Zurek (LANL), M. Saraceno (CNEA), D. Mazzitelli (UBA), D. Dalvit (LANL), J. Anglin (MIT), R. Laflamme (IQC), D. Cory (MIT), G. Morigi (UAB), S. Fernandez-Vidal (UAB), F. Cucchietti (LANL),*

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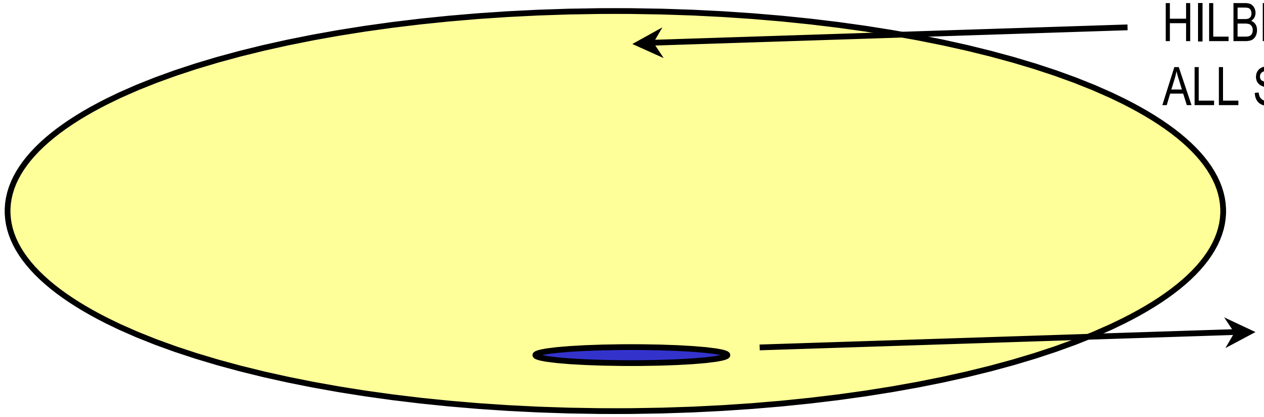
Review: J.P.Paz and W. Zurek quant-ph/0010011.

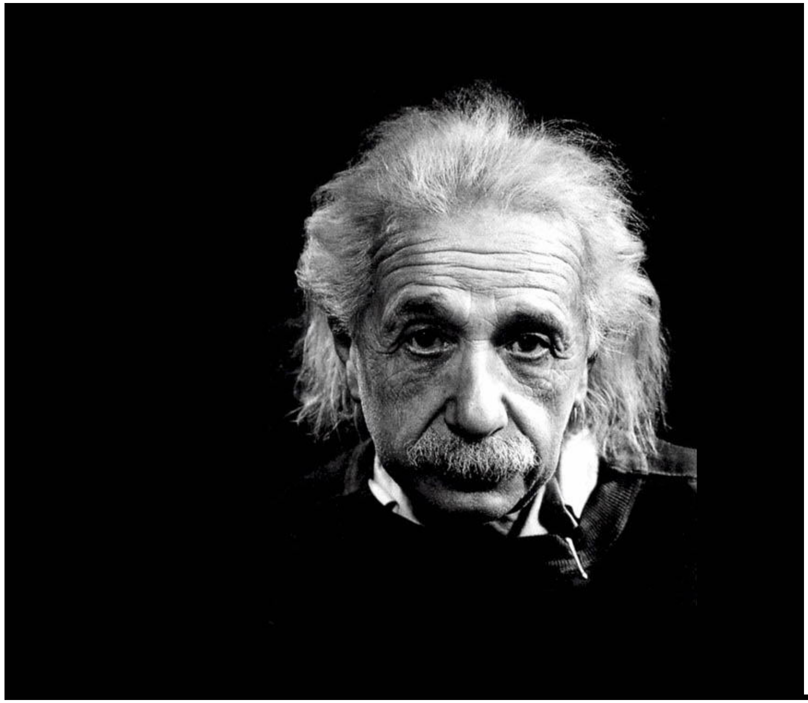
# DECOHERENCE: AN OVERVIEW

- DECOHERENCE AND THE QUANTUM-CLASSICAL TRANSITION: THE PROBLEM



HILBERT SPACE IS HUGE!!: ALL STATES ARE ALLOWED

CLASSICAL STATES: A (VERY!) SMALL SUBSET



**Albert Einstein (1954) in a letter to Max Born:**

***"Let $\varphi_1$ and $\varphi_2$ be two solutions of the same Scrodinger equation. When the system is a macro - system and when $\varphi_1$ and $\varphi_2$ are "narrow" with respect to the macro coordinates then in by far the largest number of cases this is no longer true for $(\varphi_1 + \varphi_2)/\sqrt{2}$ . Narrowness with respect to macro - coordinates is not only independent of the principles of quantum mechanics but, moreover, it is incompatible with them."***

# DECOHERENCE: AN OVERVIEW

- HOW TO EXPLAIN THE ORIGIN OF A CLASSICAL WORLD FROM A QUANTUM SUBSTRATE?: WHY IS IT THAT SOME SYSTEMS ARE ALWAYS FOUND IN “CLASSICAL STATES”? (“NARROW” WITH RESPECTO TO MACRO-COORDINATES)

- DECOHERENCE PARADIGM: CLASSICALITY IS AN EMERGENT PROPERTY

- DECOHERENCE: DYNAMICAL SUPRESSION OF QUANTUM SUPERPOSITIONS. MOST STATES ARE HIGHLY UNSTABLE. A PREFERRED SET OF STABE STATES EMERGES (IT IS DINAMICALLY SELECTED BY THE ENVIRONMENT).

- CLASSICALITY IS INDUCED ON SUBSYSTEMS BY THE ENVIRONMENT

- BASIC PHYICAL IDEA BEHIND DECOHERENCE IS VERY SIMPLE: SYSTEM-ENVIRONMENT INTERACTION CREATES CORRELATIONS

- DECOHERENCE IS INDUCED BY THE CONTINUOUS MONITORING BY THE ENVIRONMENT: A RECORD OF THE RELATIVE STATE OF THE SYSTEM IS IMPRINTED IN THE ENVIRONMENT.

- ISN’T THIS TOO SIMPLE? (HOW MUCH CAN WE BUY WITH THIS SIMPLE IDEA?)

- ENOUGH TO UNDERSTAND THE ORIGIN OF CLASSICAL FROM QUANTUM

# DECOHERENCE: AN OVERVIEW

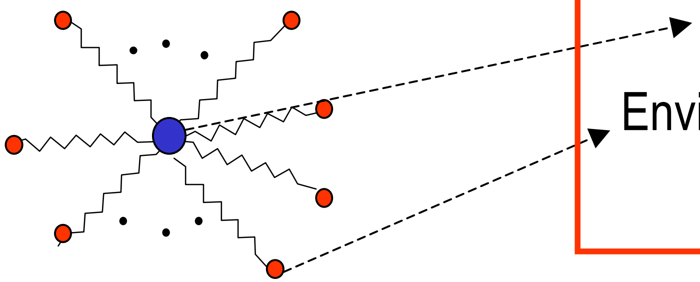
LAST DECADE: MANY QUESTIONS ON DECOHERENCE WERE ADDRESSED AND ANSWERED

- NATURE OF POINTER STATES: QUANTUM SUPERPOSITIONS DECAY INTO MIXTURES WHEN QUANTUM INTERFERENCE IS SUPRESSED. WHAT ARE THE STATES SELECTED BY THE INTERACTION? POINTER STATES: THE MOST STABLE STATES OF THE SYSTEM, DYNAMICALLY SELECTED BY THE ENVIRONMENT: W.Zurek, S. Habib & J.P. Paz, PRL 70, 1187 (1993), J.P. Paz & W. Zurek, PRL 82, 5181 (1999)
- TIMESCALES: HOW FAST DOES DECOHERENCE OCCURS? J.P. Paz, S. Habib & W. Zurek, PRD 47, 488 (1993), J. Anglin, J.P. Paz & W. Zurek, PRA 55, 4041 (1997)
- DECOHERENCE FOR CLASSICALLY CHAOTIC SYSTEMS: W. Zurek & J.P. Paz, PRL 72, 2508 (1994), D. Monteoliva & J.P. Paz, PRL 85, 3373 (2000).
- CONTROLLED DECOHERENCE EXPERIMENTS: S. Haroche et al (ENS) PRL 77, 4887 (1997), D. Wineland et al (NIST), Nature 403, 269 (2000), A. Zeillinger et al (Vienna) PRL 90 160401 (2003),
- ENVIRONMENT ENGENEERING: J.P. Paz, Nature 412, 869 (2001)
- ENTANGLEMENT DYNAMICS: J.P. Paz, PRL 100, 2200401 (2008)

# A MODEL: QUANTUM BROWNIAN MOTION

QUANTUM BROWNIAN MOTION (QBM): Paradigmatic model for a quantum open system

(realistic in many cases: Caldeira-Leggett, etc)



System: Particle (harmonic oscillator)

Environment: Collection of harmonic oscillators

Interaction: bilinear

$$H = H_S + H_E + H_{\text{int}}, \quad H_S = \frac{p^2}{2m} + V_0(x), \quad H_E = \sum_n \left( \frac{p_n^2}{2m_n} + \frac{m_n \omega_n^2}{2} q_n^2 \right), \quad H_{\text{int}} = \sum_n \lambda_n q_n x,$$

Our aim: Study evolution of the state of the system

‘State of the system’: Reduced density matrix $\rho_S = Tr_E(\rho_{SE})$

Asumption (standard): Uncorrelated initial state $t = 0$ $\rho_{SE}(0) = \rho_S(0) \otimes \rho_E(0)$

TWO “PARAMETERS”: 1) INITIAL STATE OF ENVIRONMENT (TEMPERATURE T), 2) SPECTRAL DENSITY OF ENVIRONMENT

$$J(\omega) = \sum_n \frac{\lambda_n^2}{2m_n\omega_n}\delta(\omega - \omega_n)$$

PROBLEM IS EXACTLY SOLVABLE!. USEFUL TOOL: EXACT MASTER EQUATION (EVOLUTION EQUATION FOR THE REDUCED DENSITY MATRIX); B.L. Hu, J.P. Paz and Y. Zhang, Phys. Rev. D42, 3243 (1992)

# A MODEL: QUANTUM BROWNIAN MOTION

GENERAL FORM OF THE MASTER EQUATION (VALID FOR ALL VALUES OF INITIAL TEMPERATURE OF ENVIRONMENT AND FOR ALL SPECTRAL DENSITIES)

$$\dot{\rho} = -i[H_R,\rho] - i\gamma(t)\left[x,\{p,\rho\}\right] - D(t)\left[x,[x,\rho]\right] - f(t)\left[x,[p,\rho]\right]$$

DERIVATION OF MASTER EQUATION FOR QBM (you should do this once in your lifetime!)

1) START FROM SCHROEDINGER EQUATION FOR THE SYSTEM+ENVIRONMENT

$$\dot{\rho}_T = -i\left[H_T,\rho_T\right]$$

2) GO TO THE INTERACTION PICTURE

$$H_0 = H_S + H_E \quad ; \quad U_0 = \exp\left(-iH_0 t\right) \quad ; \quad \tilde{\rho}_T = U_0^{-1}\rho_T U_0 \quad ; \quad \tilde{H}_{\text{int}} = U_0^{-1} H_{\text{int}} U_0$$

$$\dot{\tilde{\rho}}_T = -i\left[\tilde{H}_{\text{int}},\tilde{\rho}_T\right] \Rightarrow \tilde{\rho}_T(t) = \sum_{n=0}^{\infty}\int_{t_0}^{t} dt_1 \ldots \int_{t_0}^{t_{n-1}} dt_n \frac{1}{i^n}\left[\tilde{H}_{\text{int}}(t_1),\ldots,\left[\tilde{H}_{\text{int}}(t_n),\tilde{\rho}_T(t_0)\right]\right]$$

3) USE SECOND ORDER PERTURBATION THEORY

$$\tilde{\rho}_T(t) = \tilde{\rho}_T(t_0) + \frac{1}{i}\int_{t_0}^{t} dt_1\left[\tilde{H}_{\text{int}}(t_n),\tilde{\rho}_T(t_0)\right] - \int_{t_0}^{t} dt_1 \int_{t_0}^{t_2} dt_2\left[\tilde{H}_{\text{int}}(t_1),\left[\tilde{H}_{\text{int}}(t_2),\tilde{\rho}_T(t_0)\right]\right]$$

# A MODEL: QUANTUM BROWNIAN MOTION

4) TAKE TIME DERIVATIVE AND TRACE OVER THE ENVIRONMENT

$$\tilde{\rho}_T(t) = \tilde{\rho}_T(t_0) + \frac{1}{i}\int_{t_0}^{t} dt_1 \left[\tilde{H}_{\mathrm{int}}(t_n), \tilde{\rho}_T(t_0)\right] - \int_{t_0}^{t} dt_1 \int_{t_0}^{t_2} dt_2 \left[\tilde{H}_{\mathrm{int}}(t_1), \left[\tilde{H}_{\mathrm{int}}(t_2), \tilde{\rho}_T(t_0)\right]\right]$$

$$\dot{\tilde{\rho}}_T(t) = \frac{1}{i}\left[\tilde{H}_{\mathrm{int}}(t), \tilde{\rho}_T(t_0)\right] - \int_{t_0}^{t} dt_1 \left[\tilde{H}_{\mathrm{int}}(t), \left[\tilde{H}_{\mathrm{int}}(t_1), \tilde{\rho}_T(t_0)\right]\right]$$

$$\tilde{\rho} = Tr_E(\tilde{\rho}_T) \Rightarrow \dot{\tilde{\rho}}(t) = \frac{1}{i} Tr_E\left[\tilde{H}_{\mathrm{int}}(t), \tilde{\rho}_T(t_0)\right] - \int_{t_0}^{t} dt_1 Tr_E\left[\tilde{H}_{\mathrm{int}}(t), \left[\tilde{H}_{\mathrm{int}}(t_1), \tilde{\rho}_T(t_0)\right]\right]$$

5) ASSUME SYSTEM AND ENVIRONMENT ARE INITIALLY SEPARABLE

$$\dot{\tilde{\rho}}(t) = \frac{1}{i} Tr_E\left[\tilde{H}_{\mathrm{int}}(t), \tilde{\rho}(t_0) \otimes \tilde{\rho}_E(t_0)\right] - \int_{t_0}^{t} dt_1 Tr_E\left[\tilde{H}_{\mathrm{int}}(t), \left[\tilde{H}_{\mathrm{int}}(t_1), \tilde{\rho}(t_0) \otimes \tilde{\rho}_E(t_0)\right]\right]$$

6) TRICK: REPLACE THE INITIAL DENSITY MATRIX OF SYSTEM IN R.H.S. OF EQUATION!

$$\tilde{\rho}(t) = \tilde{\rho}(t_0) + \frac{1}{i}\int_{t_0}^{t} dt_1 Tr_E\left[\tilde{H}_{\mathrm{int}}(t_1), \tilde{\rho}(t_0) \otimes \tilde{\rho}_E(t_0)\right]$$

# A MODEL: QUANTUM BROWNIAN MOTION

7) NOTICE THAT THE EQUATION (VALID TO SECOND ORDER) IS LOCAL IN TIME!

$$\dot{\tilde{\rho}}(t) = \frac{1}{i} Tr_E\left[\tilde{H}_{\text{int}}(t), \tilde{\rho}(t) \otimes \tilde{\rho}_E(0)\right] - \int_0^t dt_1 Tr_E\left[\tilde{H}_{\text{int}}(t), \left[\tilde{H}_{\text{int}}(t_1), \tilde{\rho}(t) \otimes \tilde{\rho}_E(0)\right]\right[$$

$$+ \int_0^t dt_1 Tr_E\left[\tilde{H}_{\text{int}}(t), Tr_E\left(\left[\tilde{H}_{\text{int}}(t_1), \tilde{\rho}(t) \otimes \tilde{\rho}_E(0)\right]\right) \otimes \tilde{\rho}_E(0)\right]$$

8) ASSUME SIMPLE INTERACTION BETWEEN SYSTEM AND ENVIRONMENT AND GO BACK TO SCHRODINGER PICTURE FOR THE SYSTEM

$$H_{\text{int}} = \sum_k S_k \otimes E_k \qquad U_S = \exp\left(-iH_S t\right) \quad ; \quad \tilde{\rho} = U_S^{-1} \rho U_S$$

$$\dot{\rho}(t) = \frac{1}{i}\left[H_S, \rho(t)\right] + \frac{1}{i}\sum_k \left[S_k F_k, \rho(t)\right] - \frac{1}{2}\sum_{kk'} \int_0^t dt_1 \times$$

$$\times \left(-i\eta_{kk'}(t, t_1)\left[S_k, \left\{S_{k'}(t_1 - t), \rho(t)\right\}\right] + \nu_{kk'}(t, t_1)\left[S_k, \left[S_{k'}(t_1 - t), \rho(t)\right]\right]\right)$$

$$F_k = Tr_E\left(\rho_E(0) E_k\right) \quad ; \quad \eta_{kk'}(t, t') = \frac{i}{2} Tr_E\left(\rho_E(0)\left[E_k(t), E_{k'}(t')\right]\right)$$

$$\nu_{kk'}(t, t') = \frac{1}{2} Tr_E\left(\rho_E(0)\left\{E_k(t), E_{k'}(t')\right\}\right) - Tr_E\left(\rho_E(0) E_k(t)\right) Tr_E\left(\rho_E(0) E_k(t')\right)$$

# A MODEL: QUANTUM BROWNIAN MOTION

## 9) NOW CONSIDER THE QUANTUM BROWNIAN MOTION MODEL (bosonic environment)

$$H_{\text{int}} = x \otimes \sum_k \lambda_k q_k \qquad E_k(t) \to q_k(t) = q_k \cos(\omega_k t) + \frac{1}{m_k \omega_k} p_k \sin(\omega_k t)$$

$$\rho_E(0) = \exp(-H_E / k_B T)/Z \quad ; \quad n_k = \left(e^{\omega_k / k_B T} - 1\right)^{-1}$$

$$F_k = \langle q_k \rangle = 0 \quad ; \quad \eta_{kk'}(t,t') = \delta_{kk'} \frac{\lambda_k^2}{2 m_k \omega_k} \sin\left(\omega_k (t - t')\right)$$

$$\nu_{kk'}(t,t') = \delta_{kk'} \frac{\lambda_k^2}{2 m_k \omega_k} \cos\left(\omega_k (t - t')\right)\left(1 + 2 n_k\right)$$

## 10) REWRITE MASTER EQUATION ALMOST IN FINAL FORM

$$\dot{\rho}(t) = \frac{1}{i}\left[H_S, \rho(t)\right] - \frac{1}{2}\int_0^t dt_1 \left(-i\eta(t_1)\left[x,\left\{x(-t_1),\rho(t)\right\}\right] + \nu(t_1)\left[x,\left[x(-t_1),\rho(t)\right]\right]\right)$$

$$\eta(t) = \sum_k \frac{\lambda_k^2}{2 m_k \omega_k} \sin\left(\omega_k (t - t')\right) \quad ; \quad \nu(t) = \sum_k \frac{\lambda_k^2}{2 m_k \omega_k} \cos\left(\omega_k (t - t')\right)\left(1 + 2 n_k\right) \quad ; \quad J(\omega) = \sum_k \frac{\lambda_k^2}{2 m_k \omega_k} \delta(\omega - \omega_k)$$

# A MODEL: QUANTUM BROWNIAN MOTION

11) REPLACE x(t) IN THE R.H.S. OF THE MASTER EQUATION

$$\dot{\rho}(t) = \frac{1}{i}\left[H_S,\rho(t)\right] - \frac{1}{2}\int_0^t dt_1\left(-i\eta(t_1)\left[x,\left\{x(-t_1),\rho(t)\right\}\right] + \nu(t_1)\left[x,\left[x(-t_1),\rho(t)\right]\right]\right)$$

$$x(t) = x\cos(\Omega t) + \frac{1}{M\Omega}p\sin(\Omega t)$$

$$\dot{\rho} = -i\left[H_S,\rho\right] - i\frac{m}{2}\delta\omega^2(t)\left[x,\left\{x,\rho\right\}\right] - i\gamma(t)\left[x,\left\{p,\rho\right\}\right] - D(t)\left[x,\left[x,\rho\right]\right] - f(t)\left[x,\left[p,\rho\right]\right]$$

Time dependent coefficients are determined by spectral density and initial temperature

COMMENTS:

1) EQUATION WAS DERIVED TO SECOND ORDER IN THE INTERACTION BUT IT IS IN FACT AN EXACT EQUATION VALID TO ALL ORDERS.

B.L. Hu, J.P. Paz and Y. Zhang, Phys. Rev. D42, 3243 (1992)

2) IN SOME CASES ANALYTIC EXPRESSIONS FOR THE TIME DEPENDENT COEFFICIENTS CAN BE OBTAINED

# A MODEL: QUANTUM BROWNIAN MOTION

## Interpretation of the first two terms (renormalization and damping)

$$\dot{\rho} = -i\Big[\underbrace{H_S + \frac{m}{2}\delta\omega^2(t)x^2}_{\text{Dressing (renormalization)}}, \rho\Big] - \underbrace{i\gamma(t)[x,\{p,\rho\}]}_{\text{Damping (relaxation)}} - D(t)[x,[x,\rho]] - f(t)[x,[p,\rho]]$$

$$\langle \dot{p} \rangle = -m\Omega_R^2(t)\langle x \rangle - 2\gamma(t)\langle p \rangle$$

$$\langle \dot{x} \rangle = \langle p \rangle / m$$

$$\Omega_R^2(t) = \Omega^2 + \delta\omega^2(t)$$

$$\delta\omega^2(t) \approx -\frac{2}{M}\int_0^t dt' \cos(\Omega t')\eta(t')$$

$$\gamma(t) \approx \frac{1}{M\Omega}\int_0^t dt' \sin(\Omega t')\eta(t')$$

$$\eta(t) = \int_0^\infty d\omega \sin(\omega t) J(\omega)$$

- Normal friction (constant $\gamma(t)$): ohmic environment

$$J(\omega) = 2m\gamma\frac{\omega}{\pi}\frac{\Lambda^2}{\Lambda^2+\omega^2} \; ; \quad \gamma(t) = \gamma\frac{\Lambda^2}{\Lambda^2+\omega^2}\left(1-\left(\cos(\Omega t)+\frac{\Lambda}{\Omega}\sin(\Omega t)\right)e^{-\Lambda t}\right)$$

# A MODEL: QUANTUM BROWNIAN MOTION

## Interpretation of the last two terms (diffusion)

$$\dot{\rho} = -i\left[H_S + \frac{m}{2}\delta\omega^2(t)x^2, \rho\right] - i\gamma(t)\left[x,\{p,\rho\}\right] - \underbrace{D(t)\left[x,[x,\rho]\right]}_{\text{Diffusion (Decoherence)}} - \underbrace{f(t)\left[x,[p,\rho]\right]}_{\text{Anomalous Diffusion}}$$

$$\frac{d\langle p^2\rangle}{dt} = -m\Omega_R^2(t)\langle xp+px\rangle - 4\gamma(t)\langle p^2\rangle + 2D(t)$$

$$\frac{d\langle xp+px\rangle}{dt} = 2\frac{\langle p^2\rangle}{m} - 2m\Omega_R^2(t)\langle x^2\rangle - 2\gamma(t)\langle xp+px\rangle - 2f(t)$$

$$\frac{d\langle x^2\rangle}{dt} = \frac{1}{m}\langle xp+px\rangle$$

- Diffusion coefficients (D(t) and f(t)) depend on spectral density and temperature

$$D(t) \approx \int_0^t dt'\cos(\Omega t')\nu(t') \qquad f(t) \approx -\frac{1}{M\Omega}\int_0^t dt'\sin(\Omega t')\nu(t') \qquad \nu(t) = \int_0^\infty d\omega\cos(\omega t)\coth\left(\frac{\omega}{kT}\right)J(\omega)$$

$$\boxed{\begin{aligned} \langle p^2\rangle &\longrightarrow \frac{D}{2\gamma} \\ m\Omega_R^2\langle x^2\rangle &\longrightarrow \frac{D}{2m\gamma} - f \end{aligned}}$$

# A MODEL: QUANTUM BROWNIAN MOTION

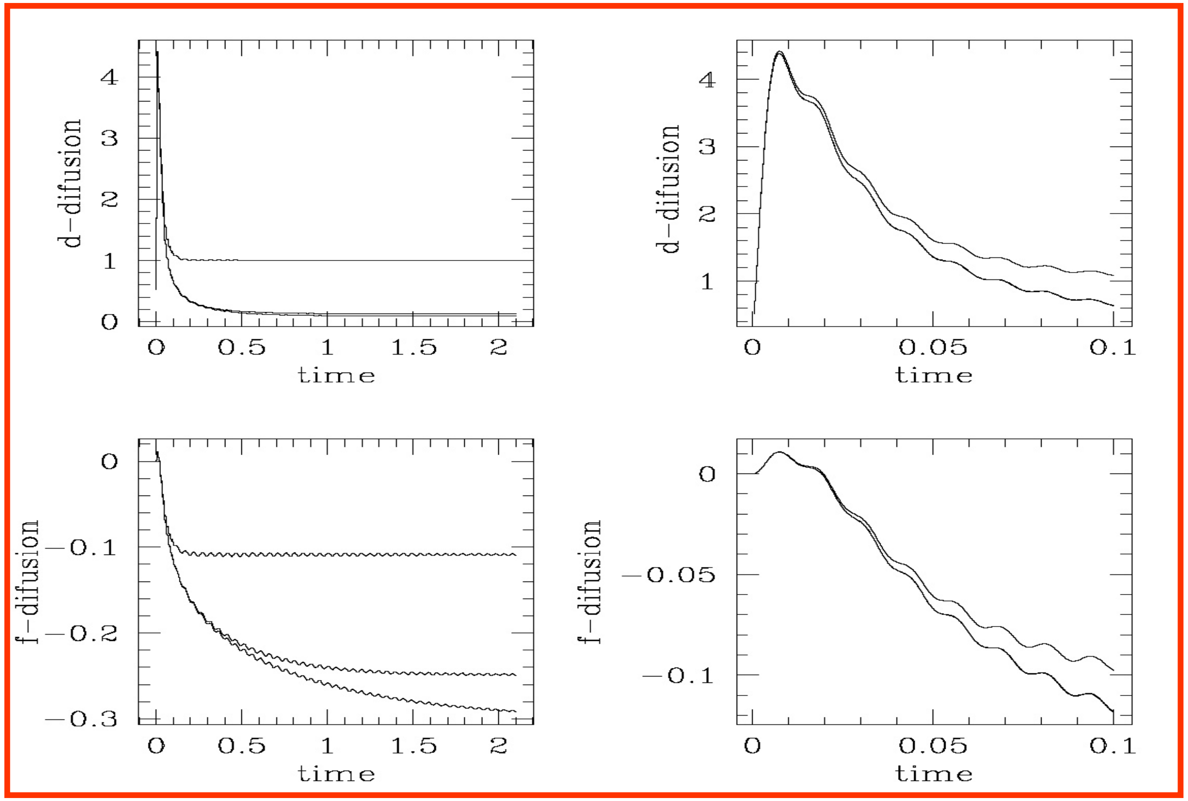
Ohmic environment

$$J(\omega) = 2m\gamma \frac{\omega}{\pi} \frac{\Lambda^2}{\Lambda^2 + \omega^2} \rightarrow 2m\gamma \frac{\omega}{\pi}; \quad \gamma(t) \rightarrow \gamma \quad \left(t >> \Lambda^{-1}\right)$$

- Diffusion coefficients (D(t) and f(t)) have initial transient and approach temperature-dependent asymptotic values

$$\langle p^2 \rangle \rightarrow \frac{D}{2\gamma}$$

$$m\Omega_R^2 \langle x^2 \rangle \rightarrow \frac{D}{2m\gamma} - f$$



# A MODEL: QUANTUM BROWNIAN MOTION

Ohmic environment in a high temperature initial state

$$\left.\begin{array}{l} J(\omega) = 2m\gamma\omega \quad (\omega \leq \Gamma), \\ k_B T >> \hbar\Omega \end{array}\right\} \quad \gamma(t) \to \gamma, \quad D(t) \to 2m\gamma k_B T, \quad f(t) \to 0$$

Approximate master equation (ohmic, high temperature)

$$\dot{\rho} = -i[H_R, \rho] - i\gamma\left[x, \{p, \rho\}\right] - D[x, [x, \rho]]$$

Use this to investigate:

1) What is the decoherence timescale?,

2) What are the pointer states?

# SUMMARY

DECOHERENCE IS AN ESENTIAL PROCESS TO UNDERSTAND THE QUANTUM CLASSICAL TRANSITION

PARADIGM: CLASSICALITY IS AN EMERGENT PROPERTY INDUCED ON SUBSYSTEMS BY THEIR ENVIRONMENTS

TOOLS TO STUDY DECOHERENCE: PHYSICS OF QUANTUM OPERN SYSTEMS

MASTER EQUATION FOR QUANTUM BROWNIAN MOTION. DERIVATION (PERTURBATIVE) AND PROPERTIES

NEXT CLASS: GENERAL RESULTS FOR LINEAR SYSTEMS (DYNAMICS AND THERMODYNAMICS)

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# PLAN

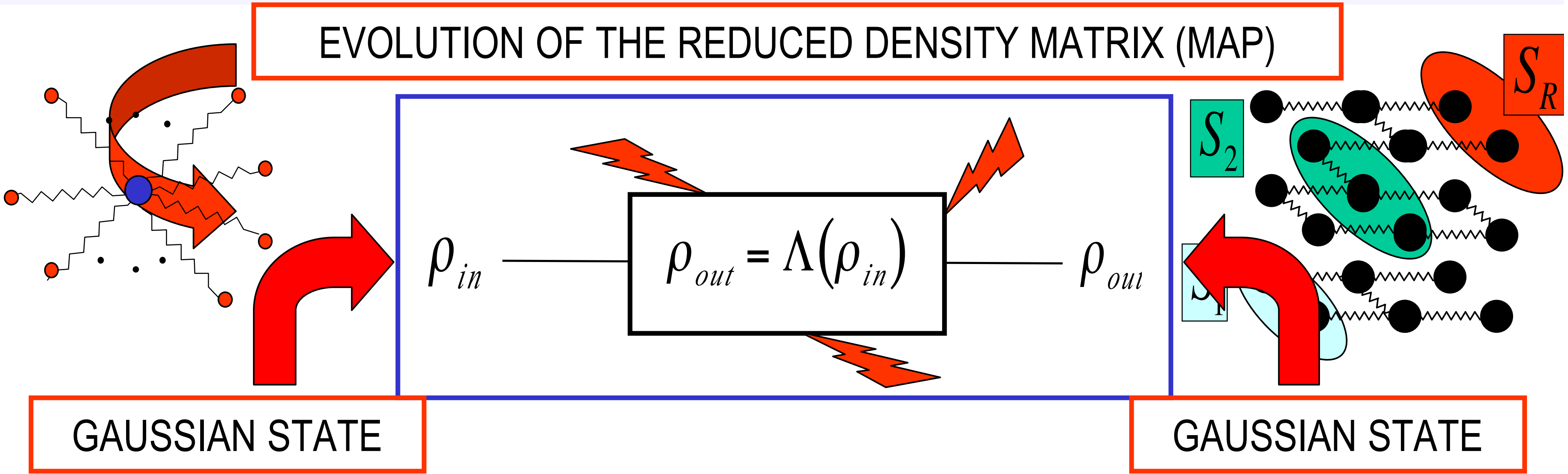
**Lecture 1: Decoherence and the quantum origin of the classical world. Evolution of quantum open systems. Quantum Brownian motion as a paradigm. Master equation.**

**Lecture 2: General results on Dynamics and Thermodynamics of linear quantum open systems. Emergence of the laws of thermodynamics. JOINT WORK WITH ESTEBAN MARTINEZ arXiv:1207.4256**

**Lecture 3: Decoherence in Quantum Brownian Motion. Decoherence timescales. Pointer states. Decoherence and disentanglement. Different dynamical phases for the behavior of quantum correlations in quantum open systems.**

**Lecture 4: Decoherence in quantum information processing. How to fight against decoherence? How to characterize decoherence? Quantum process tomography (QPT).Lecture 5: New methods for Quantum Process Tomography. “Selective and efficient QPT”. Theory and experimental implementation with single photons.**

# GENERAL RESULTS FOR LINEAR SYSTEMS



EVOLUTION OF THE REDUCED DENSITY MATRIX (MAP)

GAUSSIAN STATE

GAUSSIAN STATE

LINEAR SYSTEMS (QUADRATIC HAMILTONIANS SUCH AS QBM)

$$\Lambda(x,x';x_0,x'_0) = \langle x|\Lambda(|x_0\rangle\langle x'_0|)|x'\rangle$$

$$\rho_{out}(x,x') = \int dx_0\, dx'_0\, \Lambda(x,x';x_0,x'_0)\,\rho_{out}(x_0,x'_0)$$

LINEAR SYSTEMS: GAUSSIAN PROPAGATOR

$$\Lambda(x,x';x_0,x'_0) = \exp\left(ib_1 X\xi + ib_2 X_0\xi + ib_3 X\xi_0 + ib_4 X_0\xi_0\right)\times$$
$$\exp\left(a_1\xi^2 + a_2\xi_0^2 + a_3\xi\xi_0 + c_1X^2 + c_2X_0^2 + c_2XX_0\right)/N$$

$$X = x + x'$$
$$\xi = x - x'$$

# GENERAL RESULTS FOR LINEAR SYSTEMS

LINEAR SYSTEMS: GAUSSIAN PROPAGATOR

$$\Lambda\left(x,x';x_0,x'_0\right) = \exp\left(ib_1 X\xi + ib_2 X_0\xi + ib_3 X\xi_0 + ib_4 X_0\xi_0\right)\times$$
$$\exp\left(a_1\xi^2 + a_2\xi_0^2 + a_3\xi\xi_0 + c_1X^2 + c_2X_0^2 + c_2XX_0\right)/N$$

$$X = x + x'$$
$$\xi = x - x'$$

EXTRA CONDITIONS IMPOSE CONSTRAINTS ON COEFFICIENTS

HERMITICITY

$$\Lambda^*\left(X,\xi;X_0,\xi_0\right) = \Lambda\left(X,-\xi;X_0,-\xi_0\right)$$

$$b_1,b_2,b_3,b_4 \rightarrow real$$
$$a_1,a_2,a_3 \rightarrow real$$

TRACE PRESERVING

$$\int dX\,\Lambda\left(X,0;X_0,\xi_0\right) = \delta\left(\xi_0\right)$$

$$c_1 = c_2 = c_3 = 0$$
$$N = 2\pi/b_3$$

$$\Lambda\left(x,x';x_0,x'_0\right) = \exp\left(ib_1 X\xi + ib_2 X_0\xi + ib_3 X\xi_0 + ib_4 X_0\xi_0\right)\times$$
$$\exp\left(a_1\xi^2 + a_2\xi_0^2 + a_3\xi\xi_0\right)/2\pi b_3$$

POSITIVITY

*relations between* $a_i$ *and* $b_i$

# GENERAL RESULTS FOR LINEAR SYSTEMS

TWO MAIN RESULTS FOLLOWS DIRECTLY FROM

$$\Lambda\left(x,x';x_0,x'_0\right)=\exp\left(ib_1X\xi+ib_2X_0\xi+ib_3X\xi_0+ib_4X_0\xi_0\right)\times$$
$$\exp\left(a_1\xi^2+a_2\xi_0^2+a_3\xi\xi_0\right)/2\pi b_3$$

RESULT 1: MASTER EQUATION!

$$\dot{\rho}_{out}(x,x')=\int dx_0\,dx'_0\,\dot{\Lambda}\left(x,x';x_0,x'_0\right)\rho_{out}\left(x_0,x'_0\right)$$

$$\dot{\Lambda}\left(x,x';x_0,x'_0\right)=\left(i\dot{b}_1X\xi+i\dot{b}_2X_0\xi+i\dot{b}_3X\xi_0+i\dot{b}_4X_0\xi_0+\dot{a}_1\xi^2+\dot{a}_2\xi_0^2+\dot{a}_3\xi\xi_0-\frac{\dot{N}}{N}\right)\Lambda\left(x,x';x_0,x'_0\right)$$

$$\frac{\partial\Lambda}{\partial X}=\left(ib_1\xi+ib_3\xi_0\right)\Lambda$$
$$\frac{\partial\Lambda}{\partial\xi}=\left(ib_1X+ib_2X_0+2a_2\xi+a_3\xi_0\right)\Lambda$$

MASTER EQUATION!!!!

# GENERAL RESULTS FOR LINEAR SYSTEMS

THEOREM: GAUSSIANITY (LINEARITY), HERMITICITY AND PRESERVATION OF TRACE IMPLY THAT THERE IS A MASTER EQUATION

$$\dot{\rho}=-i\left[H_R,\rho\right]-i\gamma(t)\left[x,\{p,\rho\}\right]-i\tilde{\gamma}(t)\left[p,\{x,\rho\}\right]-$$

$$-D(t)\left[x,[x,\rho]\right]-\tilde{D}(t)\left[p,[p,\rho]\right]-f(t)\left[x,[p,\rho]\right]$$

$$H_R=\frac{p^2}{2M_R(t)}+\frac{1}{2}M_R(t)\Omega_R^2(t)x^2$$

COEFFICIENTS OF THE MASTER EQUATION DEPEND ON THOSE OF THE PROPAGATOR (i.e., a’s and b’s).

DETERMINED BY MICROSCOPIC MODELS

QUANTUM BROWNIAN MOTION (QBM) → $\tilde{\gamma}(t)=0=\tilde{D}(t);\ M_R(t)=m$

RESULT 2: EXACT SOLUTION (SIMPLE)

# GENERAL RESULTS FOR LINEAR SYSTEMS

$$\Lambda\left(x,x';x_0,x'_0\right) = \exp\left(ib_1 X\xi + ib_2 X_0\xi + ib_3 X\xi_0 + ib_4 X_0\xi_0\right)\times$$
$$\exp\left(a_1\xi^2 + a_2\xi_0^2 + a_3\xi\xi_0\right)/2\pi b_3$$

EXACT SOLUTION: SIMPLE EXPRESSION FOR CHRACTERISTIC FUNCTION

$$\chi(\alpha,t) = Tr\left(\hat{D}(\alpha)\rho(t)\right);\ \ \alpha = \begin{pmatrix} p \\ q \end{pmatrix};\ \ \hat{D}(\alpha) = \exp\left(i\left(p\hat{Q} - q\hat{P}\right)/\hbar\right)$$

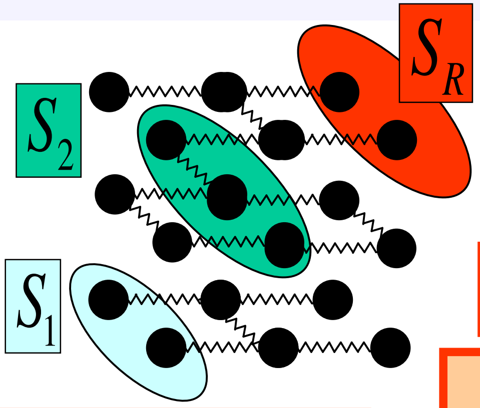
$$\chi(\alpha,t) = \chi\left(\Phi(t)\alpha,0\right)\times\exp\left(-\frac{1}{2}\alpha^T\Sigma(t)\alpha\right)$$

$$\Phi(t) = \begin{pmatrix} \Phi_{pp}(t) & \Phi_{pq}(t) \\ \Phi_{qp}(t) & \Phi_{qq}(t) \end{pmatrix};\ \ \Sigma(t) = \begin{pmatrix} \Sigma_{pp}(t) & \Sigma_{pq}(t) \\ \Sigma_{qp}(t) & \Sigma_{qq}(t) \end{pmatrix}$$

RESULTS 1 AND 2 ARE VERY GENERAL

VALID FOR ARBITRARY LINEAR NETWORKS COUPLED WITH
ARBITRARY BOSONIC RESERVOIRS!!

# GENERAL LINEAR NETWORK



$$H_S = \sum_{j=1}^{N}\left(\frac{p_j^2}{2m} + \sum_{i=1}^{N}\frac{1}{2}x_i V_{ij} x_j\right)$$

HAMILTONIAN OF NETWORK

$$H_{SE} = \sum_{e=1}^{R}\sum_{i\in S_e}\sum_{k=1}^{M} c_{k,i}^{(e)} x_i r_{k,e}$$

1

EACH ENVIRONMENT INTERACTS WITH A DIFFERENT REGION OF THE SYSTEM

$$H_E = \sum_{e=1}^{R} H_{E_e} = \sum_{e=1}^{R}\sum_{k=1}^{M}\left(\frac{\pi_{k,e}^2}{2m_k} + \frac{1}{2}m_k\omega_k^2 r_{k,e}^2\right)$$

HAMILTONIAN OF THE R-ENVIRONMENTS

EACH ENVIRONMENT HAS DIFFERENT SPECTRAL DENSITY AND TEMPERATURE

$$I_{ij}(\omega) = \sum_{e=1}^{R} I_{ij}^{(e)}(\omega) = \sum_{e=1}^{R}\sum_{k=1}^{M}\frac{c_{k,i}^{(e)}c_{k,j}^{(e)}}{2m_k\omega_k}\delta(\omega-\omega_k)$$

11

$$T_e;\ e = 1,\dots,R$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_N \end{pmatrix};\ \vec{p} = \begin{pmatrix} p_1 \\ p_2 \\ . \\ . \\ p_N \end{pmatrix}$$

$N \times N$ *matrices,* $N$ = *number of degrees of freedom*

# GENERAL RESULTS FOR LINEAR SYSTEMS

## MASTER EQUATION FOR GENERALIZED QBM

$$\dot{\rho} = -i\left[H_R,\rho\right] - i\gamma_{ij}(t)\left[x_i,\left\{p_j,\rho\right\}\right] - D_{ij}(t)\left[x_i,\left[x_j,\rho\right]\right] - f_{ij}(t)\left[x_i,\left[p_j,\rho\right]\right]$$

$$H_R = \frac{\vec{p}^T\vec{p}}{2m} + \frac{1}{2}\vec{x}^T V_R(t)\vec{x}$$

## EXACT SOLUTION FOR FOR GENERALIZED QBM

$$\chi(\alpha,t) = \chi(\Phi(t)\alpha,0) \times \exp\left(-\frac{1}{2}\alpha^T\Sigma(t)\alpha\right)$$

$$\alpha = \begin{pmatrix}\vec{p}\\ \vec{x}\end{pmatrix}_{2N}$$

$$\Phi(t) = \begin{pmatrix}\Phi_{00}(t) & \Phi_{01}(t)\\ \Phi_{10}(t) & \Phi_{11}(t)\end{pmatrix}_{2N\times 2N} \quad ; \quad \Sigma(t) = \begin{pmatrix}\Sigma_{00}(t) & \Sigma_{01}(t)\\ \Sigma_{10}(t) & \Sigma_{11}(t)\end{pmatrix}_{2N\times 2N}$$

## EXPLICIT FORM OF THE TIME DEPENDENT COEFFICIENTS CAN BE OBTAINED

# GENERAL RESULTS FOR LINEAR SYSTEMS

NOISE AND DISIPATION KERMELS (NxN MATRICES)

$$\mu(t) = \int_0^\infty d\omega \, \frac{I(\omega)}{\omega} \cos(\omega t); \quad \nu(t) = \sum_{e=1}^{R} \int_0^\infty d\omega \; I^{(e)}(\omega) \cos(\omega t) \coth\left(\frac{\omega}{2k_B T_e}\right)$$

SOLVE INTEGRO-DIFERENTIAL EQUATION

$$\ddot{G}(t) + V_R G(t) + 2\int_0^t dt' \, \mu(t-t') \, \dot{G}(t') = 0; \qquad G(0) = 0, \dot{G}(0) = 1; \; V_R = V - 2\mu(0)$$

THEN THE COEFFICIENTS OF THE EXACT SOLUTION ARE

$$\chi(\alpha,t) = \chi(\Phi(t)\alpha,0) \times \exp\left(-\frac{1}{2}\alpha^T \Sigma(t) \alpha\right)$$

$$\alpha = \begin{pmatrix} \vec{p} \\ \vec{x} \end{pmatrix}_{2N}$$

$$\Phi(t) = \begin{pmatrix} \dot{G}(t) & \ddot{G}(t) \\ G(t) & \dot{G}(t) \end{pmatrix}_{2N\times 2N} ; \quad \Sigma(t) = \begin{pmatrix} \Sigma_{00}(t) & \Sigma_{01}(t) \\ \Sigma_{10}(t) & \Sigma_{11}(t) \end{pmatrix}_{2N\times 2N}$$

$$\Sigma_{nm}(t) = \int_0^t \int_0^t dt_1 dt_2 \; G^{(n)}(t_1) \, \nu(t_1 - t_2) \, G^{(m)}(t_2)$$

# GENERAL RESULTS FOR LINEAR SYSTEMS

VERY USEFUL TO STUDY EMERGENCE OF THERMODYNAMICAL LAWS!

SOLVE EQUATION USING LAPLACE’S TRANSFORM

$$\hat{G}(s) = \left(s^2\hat{1} + V_R + 2s\hat{\mu}(s)\right)^{-1}; \quad \hat{\mu}(s) = \int_0^\infty d\omega\, I(\omega)\frac{s^2}{s^2+\omega^2}$$

WE WILL ANALYZE THE STATIONARY REGIME (LONG TIMES)

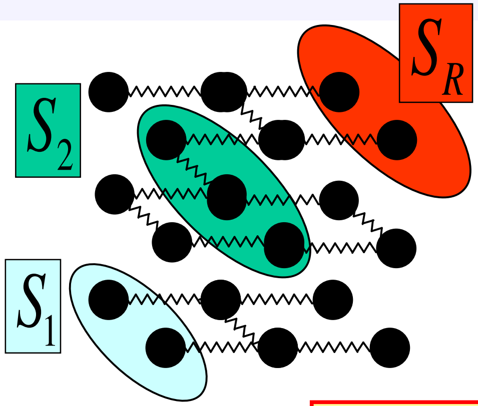
$$\chi(\alpha,t) = \chi(\Phi(t)\alpha,0)\times\exp\left(-\frac{1}{2}\alpha^T\Sigma(t)\alpha\right)\to\exp\left(-\frac{1}{2}\alpha^T\Sigma(\infty)\alpha\right)$$

$$\Phi(\infty) = \begin{pmatrix}\dot{G}(\infty) & \ddot{G}(\infty)\\ G(\infty) & \dot{G}(\infty)\end{pmatrix}_{2N\times 2N} = 0; \quad \Sigma(\infty) = \begin{pmatrix}\Sigma_{00}(\infty) & \Sigma_{01}(\infty)\\ \Sigma_{10}(\infty) & \Sigma_{11}(\infty)\end{pmatrix}_{2N\times 2N}$$

$$\Sigma_{nm}(\infty) = \Re e\left(\int_0^\infty d\omega\;\omega^{n+m} i^{n-m}\;\hat{G}(i\omega)\,\tilde{\nu}(\omega)\,\hat{G}(-i\omega)\right)$$

$$\tilde{\nu}(\omega) = \sum_{e=1}^{R} I^{(e)}(\omega)\coth\left(\frac{\omega}{2k_B T_e}\right)$$

# GENERAL LINEAR NETWORK



## ENERGY CHANGE OF SYSTEM

$$\frac{d\langle H_R\rangle^{\cdot}}{dt} = Tr\left(\dot{\rho}H_R\right) = Tr\left(V_R\Sigma_{10}(\infty)\right) = \sum_{a=1}^{R}\sum_{i\in S_a}\left(V_R\Sigma_{10}(\infty)\right)_{ii}$$

## FIRST LAW OF THERMODYNAMICS

$$\frac{d\langle H_R\rangle^{\cdot}}{dt} = \sum_{a=1}^{R}\dot{Q}_a;\;\; \dot{Q}_a = Tr\left(P_{S_a}V_R\Sigma_{10}(\infty)\right);\;\; \Sigma_{10}(\infty) = \left\langle xp^T\right\rangle$$

$$\dot{Q}_a = Heat\ \ from\ S_a\ to\ system;\;\; P_{S_a} = \mathrm{Pr}ojector\ onto\ S_a$$

$$\dot{Q}_a = \sum_{b=1}^{R}\int_0^\infty d\omega\,\omega\,Q_{a,b}(\omega)\coth\left(\frac{\omega}{2k_BT_a}\right)$$

$$Q_{a,b}(\omega) = Heat\ transfer\ matrix$$

$$Q_{a,b}(\omega) = \Im m\left(Tr\left(P_{S_a}V_RG(i\omega)I^{(b)}G(-i\omega)\right)\right)$$

$$Q_{a,b}(\omega) = -2\pi Tr\left(I^{(b)}G(i\omega)I^{(b)}G(-i\omega)\right)$$

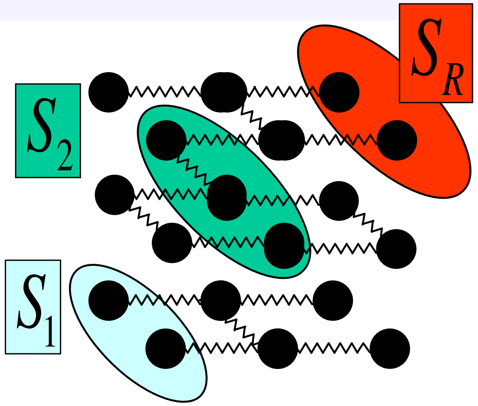
→

$$\dot{Q}_{a,b}(\omega) \le 0;\;\; a \ne b$$

$$\dot{Q}_{a,b}(\omega) = \dot{Q}_{b,a}(\omega)$$

$$\sum_{a=1}^{R}\dot{Q}_{a,b}(\omega) = 0$$

# GENERAL LINEAR NETWORK



SECOND LAW: HEAT FLOWS FROM HOT TO COLD

$$\sum_{a=1}^{R} \dot{Q}_{a,b}(\omega) = 0 \Rightarrow$$

$$\dot{Q}_a = \int_0^{\infty} d\omega\, \omega \sum_{b \neq a}^{R} Q_{a,b}(\omega)\left(\coth\left(\frac{\omega}{2k_B T_b}\right) - \coth\left(\frac{\omega}{2k_B T_a}\right)\right)$$

$$if\ T_a \geq T_b, \forall\, b \Rightarrow \dot{Q}_a \geq 0!!!; \quad if\ T_a \leq T_b, \forall\, b \Rightarrow \dot{Q}_a \leq 0!!!;$$

$$also\ \frac{dS}{dt} = \sum_{e=1}^{R} \frac{\dot{Q}_a}{T_a} \leq 0$$

HOTTEST RESERVOIR ALWAYS INJECTS ENERGY (COOLS DOWN)

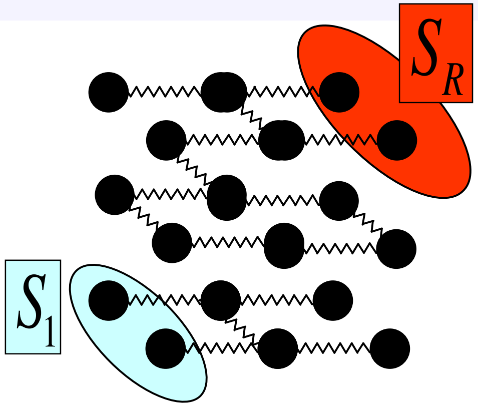
COLDEST RESERVOIR ALWAYS EXTRACTS ENERGY (HEATS UP)

RESULTS CAN BE GENERALIZED:

1) MULTIPLE ENVIRONMENTS IN EACH REGION
2) OVERLAPPING REGIOS

NO-GO THEOREM FOR LINEAR QUANTUM ABSORPTION REFRIGERATOR!! (QAR were proposed by Kosloff et al -PRL, 2012- using nonlinear system: non-linearity is essential!!!)

# GENERAL LINEAR NETWORK



THIRD LAW (UNATTAINABILITY OF ZERO TEMPERATURE)

SIMPLEST CASE: TWO RESERVOIRS WITH IDENTICAL SPECTRAL DENSITIES

$$I^{(1,2)}(\omega) = \gamma\omega^P \;\theta(\omega)\; M^{(1,2)}_{N\times N}$$

LOW TEMPERATURES

$$\dot{Q}_1 = \int_0^\infty d\omega\,\omega\,Q_{1,2}(\omega)\left(\coth\left(\frac{\omega}{2k_BT_2}\right) - \coth\left(\frac{\omega}{2k_BT_1}\right)\right)$$

$$\dot{Q}_1 = 2\pi\,\gamma^2\int_0^\infty d\omega\,\omega^{1+2P}\;\theta^2(\omega)\,\left|G(i\omega)\right|^2\left(\coth\left(\frac{\omega}{2k_BT_2}\right) - \coth\left(\frac{\omega}{2k_BT_1}\right)\right)$$

THIRD LAW IS OBTAINED FOR SUB-OHMIC (p<1), OHMIC (p=1) AND SUPER-OHMIC (p>1) ENVIRONMENTS

$$\dot{Q}_1 = C\,\overline{T}^{1+2P}\Delta T;\quad \overline{T} = \frac{T_1+T_2}{2};\quad \Delta T = T_1 - T_2$$

$$\frac{dS_1}{dt} = \frac{\dot{Q}_1}{T_1} = C'\,\overline{T}^{2P}\Delta T \to 0,\ if\ \ p \geq 0$$

# SUMMARY

GENERAL RESULTS FOR ARBITRARY LINEAR OPEN NETWORKS: MASTER EQUATION AND EXACT SOLUTION

STUDY LONG TIME STATIONARY LIMIT

DERIVE THE LAWS OF THERMODYNAMICS

NO-GO THEOREM FOR QUANTUM ABSORPTION REFRIGERATOR

THIRD LAW IMPOSES CONSTRAINT ON SPECTRAL DENSITY (LOW FREQUENCIES)

# QUANTUM OPEN SYSTEMS, DECOHERENCE AND QUANTUM PROCESS TOMOGRAPHY

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JULY 2012

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Universidad
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# PLAN

Lecture 1: Decoherence and the quantum origin of the classical world. Evolution of quantum open systems. Quantum Brownian motion as a paradigm. Master equation.

Lecture 2: General results on Dynamics and Thermodynamics of linear quantum open systems. Emergence of the laws of thermodynamics. E.A. MARTINEZ & J.P.P. arXiv:12074256

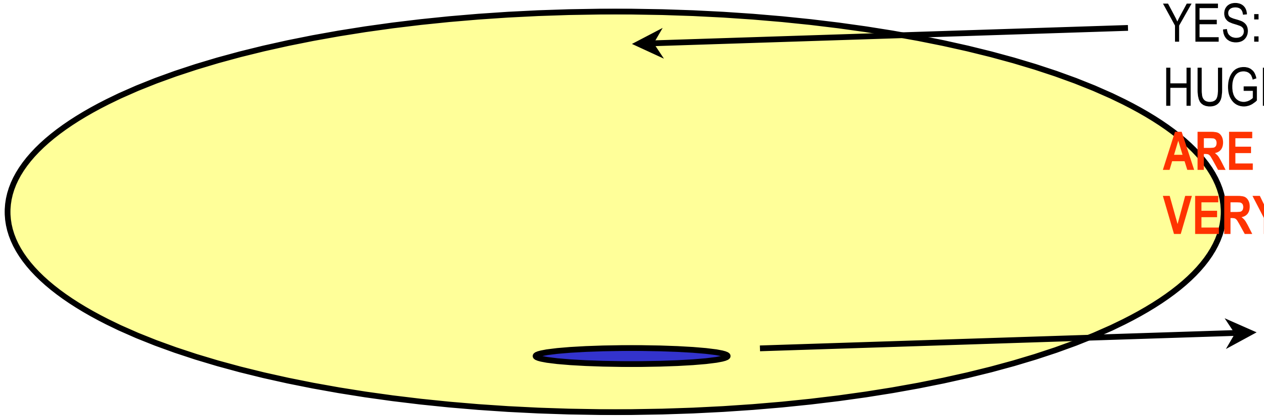
Lecture 3: Decoherence in Quantum Brownian Motion. Decoherence timescales. Pointer states. Decoherence and disentanglement. Different dynamical phases for the behavior of quantum correlations in quantum open systems.

Lecture 4: Decoherence in quantum information processing. How to fight against decoherence? How to characterize decoherence? Quantum process tomography (QPT).

Lecture 5: New methods for Quantum Process Tomography. "Selective and efficient QPT". Theory and experimental implementation with single photons.

# DECOHERENCE: AN OVERVIEW

- DECOHERENCE AND THE QUANTUM-CLASSICAL TRANSITION:



YES: HILBERT SPACE IS HUGE, **BUT MOST STATES ARE UNSTABLE!! (DECAY VERY FAST INTO MIXTURES)**

CLASSICAL STATES: A (VERY!) SMALL SUBSET. **THEY ARE THE POINTER STATES OF THE SYSTEM DYNAMICALLY CHOSEN BY THE ENVIRONMENT**

- POINTER STATES: W.Zurek, S. Habib & J.P. Paz, PRL 70, 1187 (1993), J.P. Paz & W. Zurek, PRL 82, 5181 (1999)
- TIMESCALES: J.P. Paz, S. Habib & W. Zurek, PRD 47, 488 (1993), J. Anglin, J.P. Paz & W. Zurek, PRA 55, 4041 (1997)
- CONTROLLED DECOHERENCE EXPERIMENTS: Zeillinger et al (Vienna) PRL 90 160401 (2003), Haroche et al (ENS) PRL 77, 4887 (1997), Wineland et al (NIST), Nature 403, 269 (2000).

# A MODEL: QUANTUM BROWNIAN MOTION

Ohmic environment in a high temperature initial state

$$\left.\begin{array}{l} J(\omega) = 2m\gamma\omega \quad (\omega \leq \Gamma), \\ k_B T >> \hbar\Omega \end{array}\right\} \quad \gamma(t) \to \gamma, \quad D(t) \to 2m\gamma k_B T, \quad f(t) \to 0$$

Approximate master equation (ohmic, high temperature)

$$\dot{\rho} = -i[H_R, \rho] - i\gamma[x, \{p, \rho\}] - D[x, [x, \rho]]$$

Use this to investigate:

1) What is the decoherence timescale?,

2) What are the pointer states?

# DECOHERENCE IN QUANTUM BROWNIAN MOTION

DECOHERENCE IN QUANTUM BROWNIAN MOTION: MAIN RESULTS ARE BETTER SEEN REPRESENTING THE STATE IN PHASE SPACE VIA WIGNER FUNCTIONS

$$W(x,p) = \int \frac{dy}{2\pi\hbar}\, e^{ipy/\hbar} \langle x - y/2 | \rho | x + y/2 \rangle$$

- PROPERTIES:
  - ⇨ W(x,p) is real
  - ⇨ Use it to compute inner products as: $\int dx\, dp\, W_1(x,p)\, W_2(x,p) = \frac{1}{2\pi\hbar} Tr(\rho_1\ \rho_2)$
  - ⇨ Integral along lines give all marginal distributions:

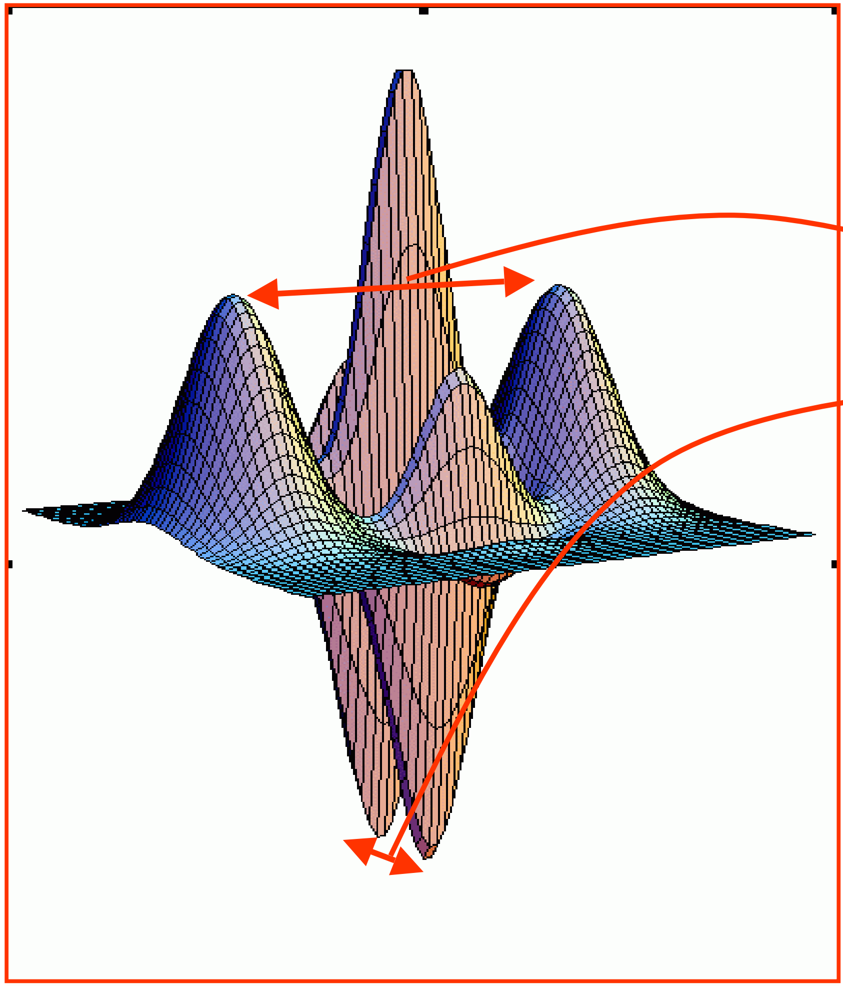
$$\int_{ax+bp=c} dx\, dp\, W(x,p) = \Pr obability(aX + bP = c)$$

MASTER EQUATION CAN BE REWRITTEN FOR THE WIGNER FUNCTION: IT HAS THE FORM OF A FOKER-PLANCK EQUATION

$$\dot{W} = \{H_0, W\}_{MB} + \gamma\partial_p(pW) + D\partial^2_{pp} W + f\partial^2_{xp} W$$

# DECOHERENCE IN QUANTUM BROWNIAN MOTION

HOW DOES THE WIGNER FUNCTION OF A QUANTUM STATE LOOK LIKE?:
SUPERPOSITION OF TWO GAUSSIAN STATES



$Dis\tan ce \quad L$

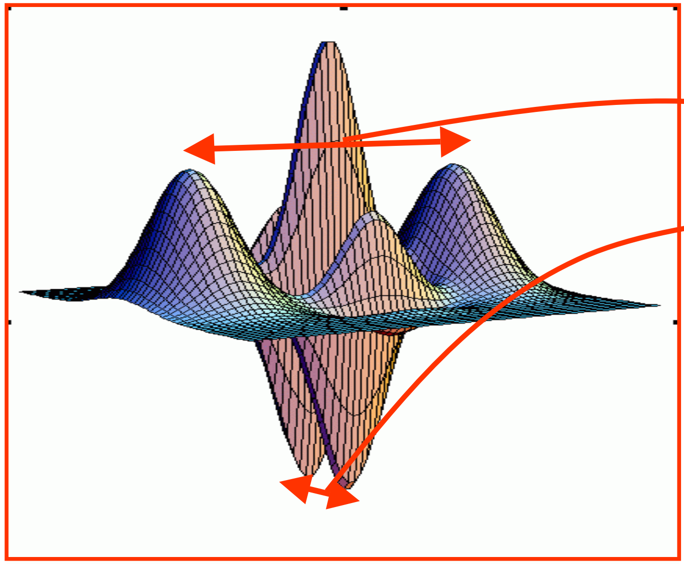
$Wavelength \quad \lambda_p = \frac{\gg}{L}$

OSCILLATIONS IN WIGNER FUNCTION: THE SIGNAL OF QUANTUM INTERFERENCE. HOW DOES DECOHERENCE AFFECTS THIS STATE?

# DECOHERENCE IN QUANTUM BROWNIAN MOTION

MASTER EQUATION CAN BE REWRITTEN FOR THE WIGNER FUNCTION: IT HAS THE FORM OF A FOKER-PLANCK EQUATION

$$\dot{W} = \{H_0, W\}_{MB} + \gamma\partial_p(pW) + D\partial^2_{pp}W + f\partial^2_{xp}W$$



$Dis\tan ce \quad L$

$Wavelength \quad \lambda_p = \frac{\gg}{L}$

$$\dot{W} = \{H_0, W\}_{MB} + D\partial^2_{pp}W + \gg$$

$$W_{osc} \approx A(t)\cos(k_p p) \Rightarrow A(t) \approx \exp(-\Gamma t)$$

$$\Gamma = Dk_p^{\ 2}$$

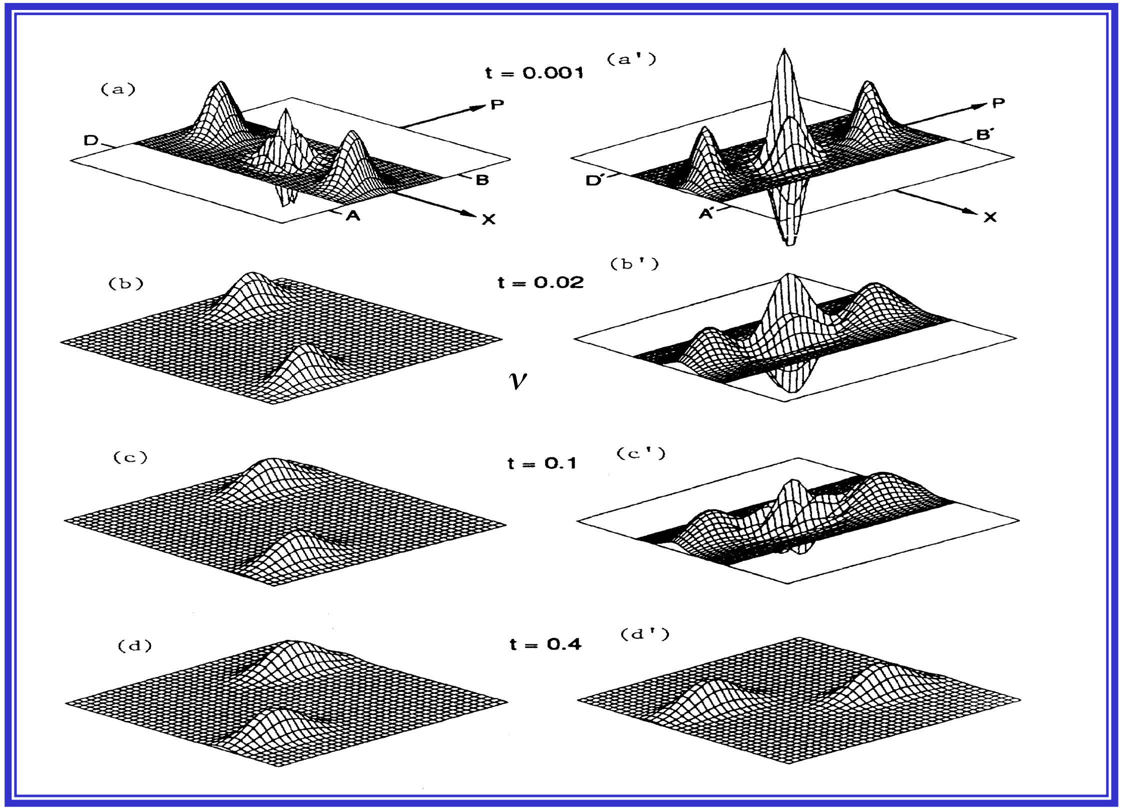
$$\Gamma = DL^2/\gg^2, \qquad D = 2m\gamma k_B T, \qquad \lambda_{DB} = \gg/\sqrt{2m\,k_B T}$$

$$\Rightarrow \Gamma = \gamma\left(L/\lambda_{DB}\right)^2 \approx 10^{40}\gamma, \quad m = 1gr,\ T = 300K,\ L = 1cm$$

DECOHERENCE RATE:
MUCH LARGER THAN
RELAXATION RATE

# POINTER STATES, DECOHERENCE TIMESCALE

EVOLUTION OF WIGNER FUNCTION



NOTICE: NOT ALL STATES ARE AFFECTED BY THE ENVIRONMENT IN THE SAME WAY (SOME SUPERPOSITIONS LAST LONGER THAN OTHERS)

# POINTER STATES, DECOHERENCE TIMESCALE

NOT ALL STATES ARE AFFECTED BY DECOHERENCE IN THE SAME WAY

QUESTION: WHAT ARE THE STATES WHICH ARE MOST ROBUST UNDER DECOHERENCE?

POINTER STATES: STATES WHICH ARE MINIMALLY AFFECTED BY THE INTERACTION WITH THE ENVIRONMENT

AN OPERATIONAL DEFINITION OF POINTER STATES:

“PREDICTABILITY SIEVE”

$$|\Psi(0)\rangle\langle\Psi(0)| \xrightarrow{\text{t}} \rho(t)$$

Initial state of the system (pure)

State of system at time t (mixed)

$$S_{VN}(t) = -Tr\big(\rho(t)\ln(\rho(t))\big), \quad \xi(t) = Tr\big(\rho^2(t)\big)$$

Measure degradation of system’s state with entropy (von Neuman) or purity decay

# POINTER STATES, DECOHERENCE TIMESCALE

$$\boxed{S_{VN}(t) = -Tr\big(\rho(t)\ln(\rho(t))\big), \quad \zeta(t) = Tr\big(\rho^2(t)\big)}$$

These quantities depend on time AND on the initial state

PREDICTABILITY SIEVE: FIND THE INITIAL STATES SUCH THAT THESE QUANTITIES ARE MINIMIZED (FOR A DYNAMICAL RANGE OF TIMES)

PREDICTABILITY SIEVE IN A PHYSICALLY INTERESTING CASE?

ANALIZE QUANTUM BROWNIAN MOTION

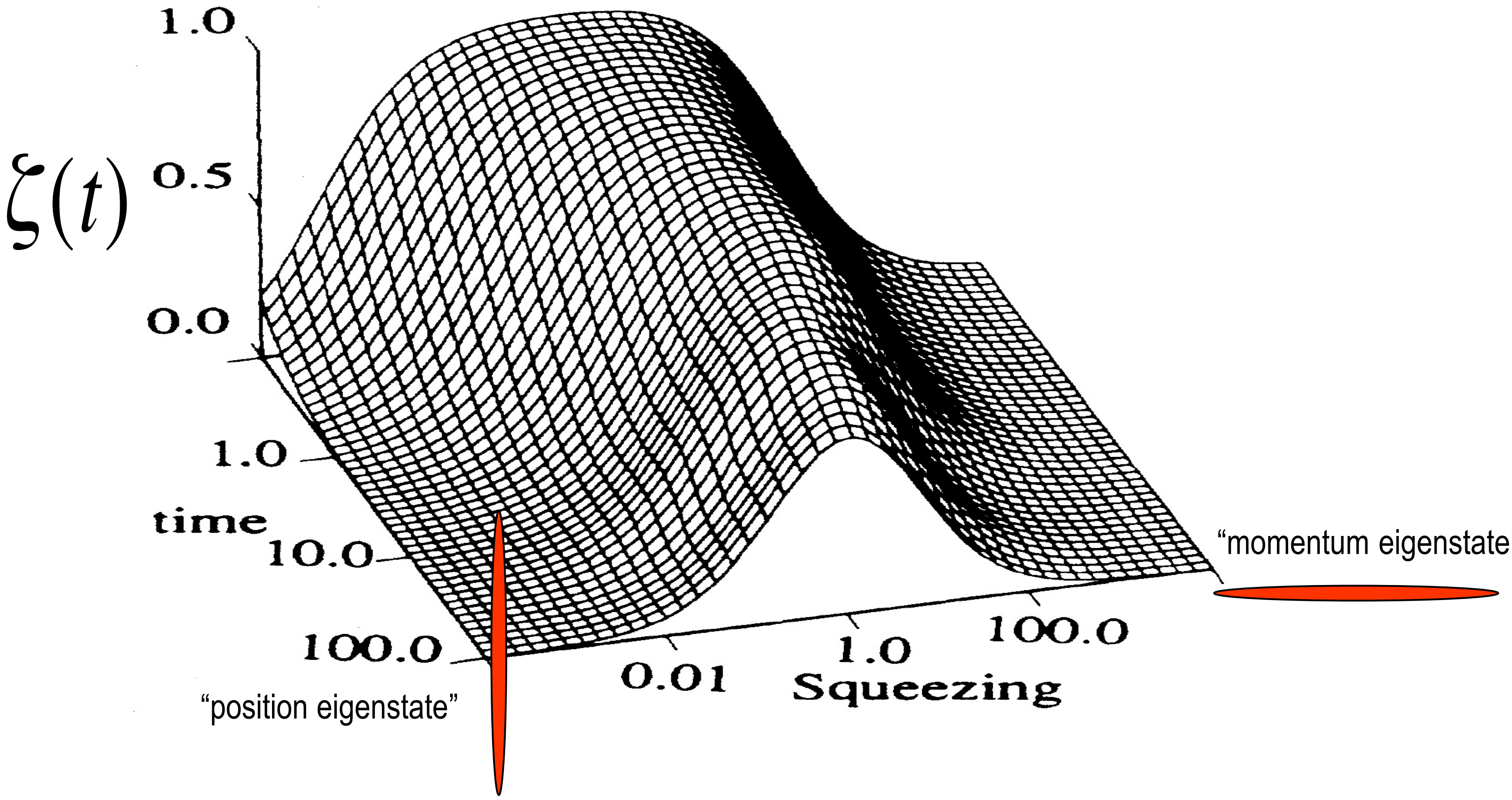
USE MASTER EQUATION TO ESTIMATE PURITY DECAY OR ENTROPY GROWTH

$$\dot{\rho} = -i\left[H_R + \frac{m}{2}\delta\omega^2(t)x^2, \rho\right] - i\gamma(t)[x,\{p,\rho\}] - D(t)[x,[x,\rho]] - f(t)[x,[p,\rho]]$$

$$\boxed{\dot{\zeta} = 2Tr(\dot{\rho}\rho) = 2\gamma\zeta - 2DTr\left([x,\rho]^2\right) + 2fTr\left([x,\rho][p,\rho]\right)}$$

# POINTER STATES, DECOHERENCE TIMESCALE

A SIMPLE SOLUTION FROM THE PREDICTABILITY SIEVE CRITERION



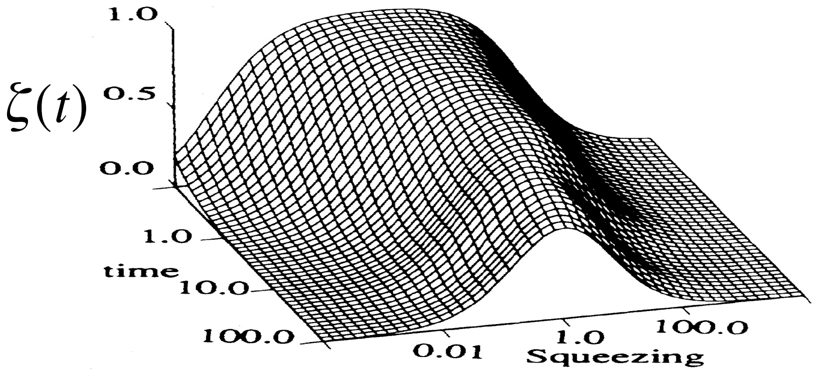
Minimize over initial state: Pointer states for QBM are minimally uncertainty coherent states!
W.Zurek, J.P.P & S. Habib, PRL 70, 1187 (1993)

# POINTER STATES, DECOHERENCE TIMESCALE

WARNING: DIFFERENT POINTER STATES IN DIFFERENT REGIMES!

1) Dynamical regime (QBM): Pointer basis results from interplay between system and environment

2) “Slow system” regime: (Quantum Measurement): System’s evolution is negligible, Pointer basis is determined by the interaction Hamiltonian (position in QBM)



3) “Slow environment” regime: The evolution of the environment is very “slow” (adiabatic environment): Pointer states are eigenstates of the Hamiltonian of the system! The environment only “learns” about properties of system which are non-vanishing when averaged in time. J.P. Paz & W.Zurek, PRL 82, 5181 (1999)

TAILOR MADE POINTER STATES? Environmental engeneering…

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**Lecture 3: Decoherence in Quantum Brownian Motion. Decoherence timescales. Pointer states. Decoherence and disentanglement. Different dynamical phases for the behavior of quantum correlations in quantum open systems.**

**Lecture 4: Decoherence in quantum information processing. How to fight against decoherence? How to characterize decoherence? Quantum process tomography (QPT).**

**Lecture 5: New methods for Quantum Process Tomography. "Selective and efficient QPT". Theory and experimental implementation with single photons.**

ENTANGLEMENT IS A RESOURCE

WHY DON’T WE HAVE QUANTUM TECHNOLOGIES (ENTANGLEMENT BASED) AROUND US?

DUE TO DECOHERENCE
(induces disentanglement, quantum-classical transition)

HOW DOES ENTANGLEMENT BEHAVE IN A QUANTUM OPEN SYSTEM? DOES IT DISSAPEAR? CAN IT PERSIST FOR LONG TIMES?…

**“Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?**
*A. Einstein, B. Podolsky, N. Rosen.*
Physical Review **47**, 1935, 777-780.

## Entangled states of two particles (two modes)

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \; ; \vec{P} = \vec{P}_1 + \vec{P}_2$$

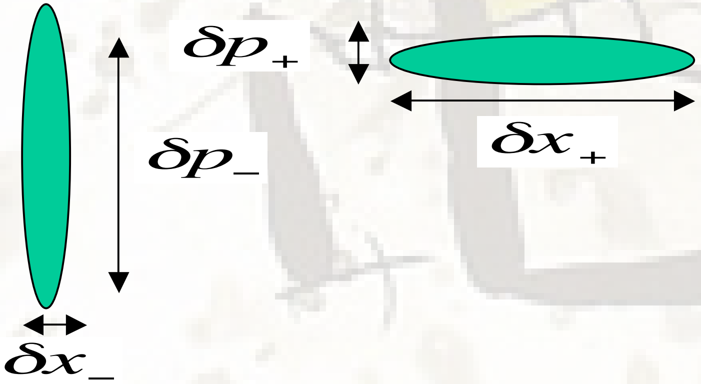
$$|\Psi\rangle_{12} = \left|\vec{r} = \vec{r}_0 \; ; \vec{P} = \vec{P}_0\right\rangle$$

$$\Psi\left(\vec{r},\vec{P}\right) = \delta(\vec{r} - \vec{r}_0)\delta\left(\vec{P} - \vec{P}_0\right)$$

**These are non-physical (idealized) states**
**A set of physical (yet entangled) states: Two mode squeezed states.**

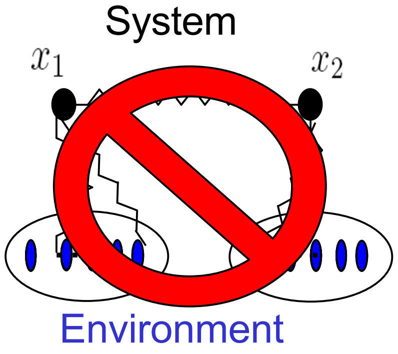
$$a_{\substack{1\\2}} = x_{\substack{1\\2}} + i\, p_{\substack{1\\2}} \; ; \quad |\Psi\rangle_{12} = \exp\left(r\left(a_1{}^{+}a_2{}^{+} - a_1 a_2\right)\right)|0\rangle_{12}$$

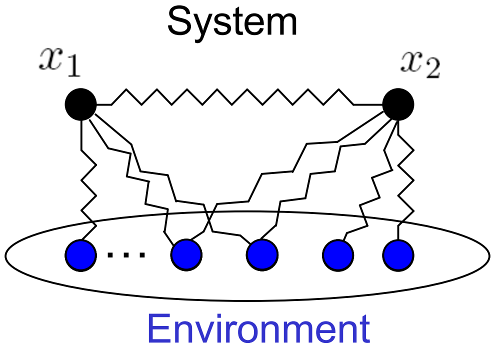
$$\frac{m\Omega\delta x_-}{\delta p_-} = \frac{\delta p_+}{m\Omega\delta x_+} = \exp(2r)$$

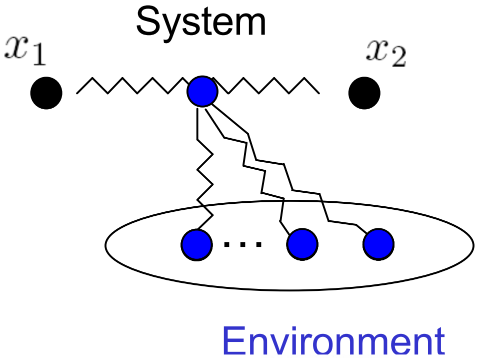


**Entangled Gaussian states**
**Measure of entanglement**
**E=2r**
**Entanglement can be transfered to spins by local operations and then it can be used…**

# A SIMPLE MODEL WITH NON-TRIVAL PHASES Two oscillators & a common reservoir. Long time evolution of entanglement.







PRL **100,** 220401 (2008) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2008

### Dynamics of the Entanglement between Two Oscillators in the Same Environment

Juan Pablo Paz and Augusto J. Roncaglia
*Departamento de Física, FCEyN, UBA, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
(Received 19 November 2007; published 2 June 2008)

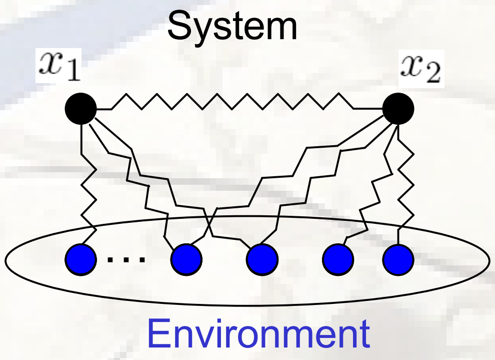
# A PROPOSED ION TRAP EXPERIMENT

PHYSICAL REVIEW A **81**, 022306 (2010)

### Observing different phases for the dynamics of entanglement in an ion trap

Cecilia Cormick and Juan Pablo Paz
*Departamento de Física, FCEyN, UBA, & IFIBA CONICET, Ciudad Universitaria Pabellón 1, 1428 Buenos Aires, Argentina*
(Received 24 August 2009; published 9 February 2010)

# THE MODEL:



# THE ENTANGLEMENT MEASURE

SIMPLICITY: GAUSSIAN INITIAL STATES (GOOD ENTANGLEMENT MEASURE)

○

$$V_{ij}(t)=\frac{1}{2}\left\langle\left\{r_i,r_j\right\}\right\rangle-\left\langle r_i\right\rangle\left\langle r_j\right\rangle;\ \vec{r}=\left(x_1,p_1,x_2,p_2\right)$$

Entanglement measure: ***Logarithmic Negativity***

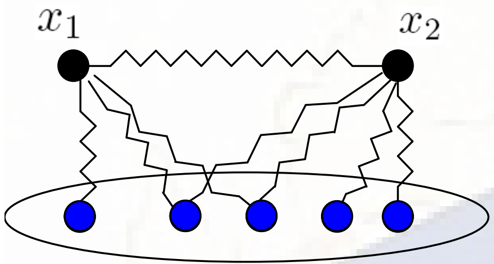
○

$$E_N=\max\left\{0,-\ln\left(2\nu_{\min}\right)\right\}$$

$$\nu_{\min}=\textit{lowest symplectic eigenvalue of partially transposed } V_{ij}$$

$\nu_{\min}$

# THE SOLUTION

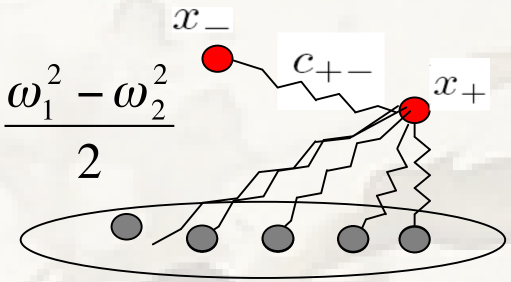


Environment

Symmetric coupling

Resonant oscillators

$$c_{+-} = \frac{\omega_1^2 - \omega_2^2}{2}$$



Resonant oscillators

ALMOST IDENTICAL TO THE USUAL “QUANTUM BROWNIAN MOTION” MODEL

INTEGRATING OUT (TRACING OUT) THE ENVIRONMENT WE OBTAIN AN EXACT MASTER EQUATION

$$\dot{\rho} = -i[H_R,\rho] - i\gamma(t)[x_+,\{p_+,\rho\}] - D(t)[x_+,[x_+,\rho]] - f(t)[x_+,[p_+,\rho]]$$

Renormalized Hamiltonian

Damping term

Normal diffusion

Anomalous diffusion

Ohmic environment

$$J(\omega) = \sum_n \frac{\lambda_n^2}{2m_n\omega_n}\delta(\omega-\omega_n) = 2m\gamma\,\frac{\omega}{\pi}\theta(\Lambda-\omega)\,; \quad \gamma(t)\rightarrow\gamma \quad \left(t >> \Lambda^{-1}\right)$$

$high \quad T$

$D \rightarrow 2m\gamma T$

$f \rightarrow -\frac{2\gamma T}{\pi\Omega}\log\frac{\Lambda+\Omega}{\Lambda-\Omega}$

- Asymptotic regime: $\Delta^2 p_+ = \langle p_+^2\rangle, \quad \Delta^2 x_+ = \langle x_+^2\rangle$

$$\Delta p_+ = \sqrt{\frac{D}{2\gamma}}; \quad \Omega\Delta x_+ = \sqrt{\frac{D}{2m^2\gamma} - \frac{f}{m}}$$

$T = 0$

$D \rightarrow m\gamma\Omega + \frac{2m\gamma^2}{\pi}\left(2\log\frac{\Lambda}{\Omega} - 1\right)$

$f \rightarrow \frac{2\gamma}{\pi}\log\frac{\Lambda}{\Omega}$

# REMEMBER: A PROPERTY OF THE SOLUTION

CONSIDER THE ASYMPTOTIC STATE FOR QUANTUM BROWNIAN MOTION

$$\dot{\rho} = -i[H_R,\rho] - i\gamma(t)[x_+,\{p_+,\rho\}] - D(t)[x_+,[x_+,\rho]] - f(t)[x_+,[p_+,\rho]]$$

$$\Delta p = \sqrt{\frac{D}{2\gamma}}; \qquad \Omega_R \Delta x = \sqrt{\frac{D}{2m^2\gamma} - \frac{f}{m}}$$

EXACT MASTER EQUATION B.L. Hu, J.P. Paz and Y. Zhang, Phys. Rev. D42, 3243 (1992)

$$\langle K \rangle = \frac{\langle p^2 \rangle}{2m} = \frac{D}{4m\gamma}$$

$$\langle V \rangle = \frac{1}{2} m \Omega_R^2 \langle x^2 \rangle = \frac{D}{4m\gamma} - \frac{f}{2}$$

$$r_c = \frac{1}{2}\log\left(\frac{m\Omega_R \Delta x}{\Delta p}\right) = \frac{1}{4}\log\left(1 - \frac{2m\gamma f}{D}\right)$$

ASYMPTOTIC STATE VIOLATES EQUIPARTITION!

ASYMPTOTIC STATE IS SQUEEZED!!

QUANTUM WEIRDNESS??

FOLLOWS FROM MASTER EQUATION… COEFFICIENT f IS THE ONE TO BLAME!

# COMPUTING ENTANGLEMENT FOR LONG TIMES

- In the asymptotic regime:

$$\Delta p_+ = \sqrt{\frac{D}{2\gamma}}; \quad \Omega\Delta x_+ = \sqrt{\frac{D}{2m^2\gamma} - \frac{f}{m}} \qquad \langle\{x_+, p_+\}\rangle = 0$$

The equilibrium state is squeezed due to $f$ (low temperature regime)

THEN THE ASYMPTOTIC FORM OF THE COVARIANCE MATRIX IS

$$V = \begin{array}{c} \begin{array}{cccc} x_- & p_- & x_+ & p_+ \end{array} \\ \left(\begin{array}{cccc} & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & \Delta^2 p_+ & 0 \\ 0 & 0 & 0 & \Delta^2 x_+ \end{array}\right) \end{array}$$

Free oscillator of mass $m$ and frequency $\Omega_-$

Correlations between the oscillators are zero in the asymptotic regime

Equilibrium moments of $x_+$ oscillator

# ENTANGLEMENT IN THE ASYMPTOTIC STATE

EVALUATE ANALYTICALLY THE LOGARITHMIC NEGATIVITY FOR THE $(x_1, x_2)$ OSCILLATORS

$$E_{\mathcal{N}}(t) \rightarrow \max\{0, \tilde{E}_{\mathcal{N}} + \Delta E_{\mathcal{N}} G(t)\}$$

$G(t)$ is a function with period $\pi/\Omega_-$ in $\{-1, 1\}$

Mean value: $$\tilde{E}_{\mathcal{N}} = \max\{r, r_{crit}\} - \frac{1}{2}\ln(4\Delta x_+ \Delta p_+ \delta x_- \delta p_-)$$

Amplitude of oscillations: $$\Delta E_{\mathcal{N}} = \min\{r, r_{crit}\}$$

Where:

$$r = |\frac{1}{2}\ln(m\Omega_- \frac{\delta x_-}{\delta p_-})|$$ → Squeezing of the $x_-$ oscillator

$$r_{crit} = |\frac{1}{2}\ln(m\Omega_- \frac{\Delta x_+}{\Delta p_+})|$$ → Squeezing of equilibrium for the oscillator $x_+$

$$\tilde{E}_N \rightarrow (Maximal\ Sqeezing) - (Entropy)$$

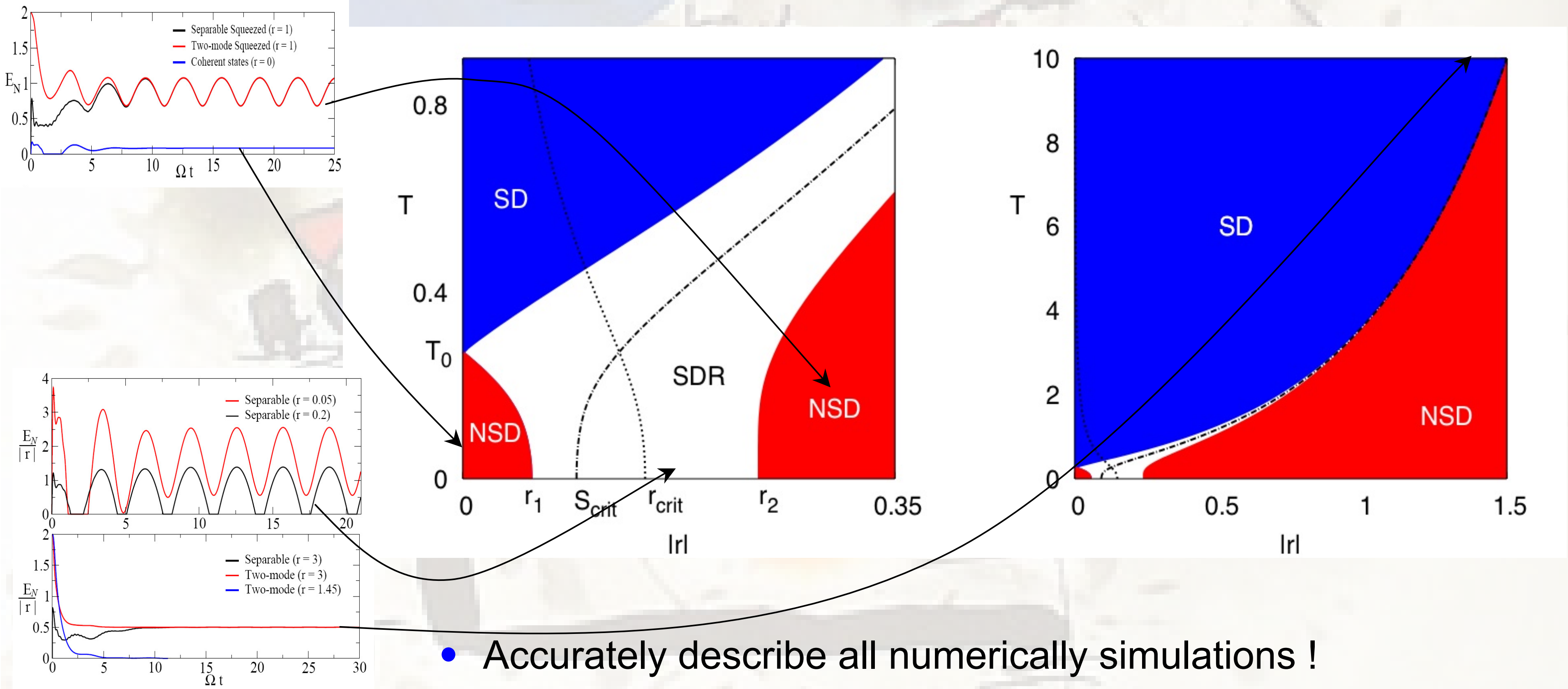
$$\Delta E_N \rightarrow (Minimal\ Sqeezing)$$

WE CONCLUDE THAT THERE ARE THREE QUALITATIVELY DIFFERENT ASYMPTOTIC BEHAVIORS!!

# THREE DYNAMICAL PHASES FOR ENTANGLEMENT

- Phase diagram (complete description of the asymptotic behavior) :

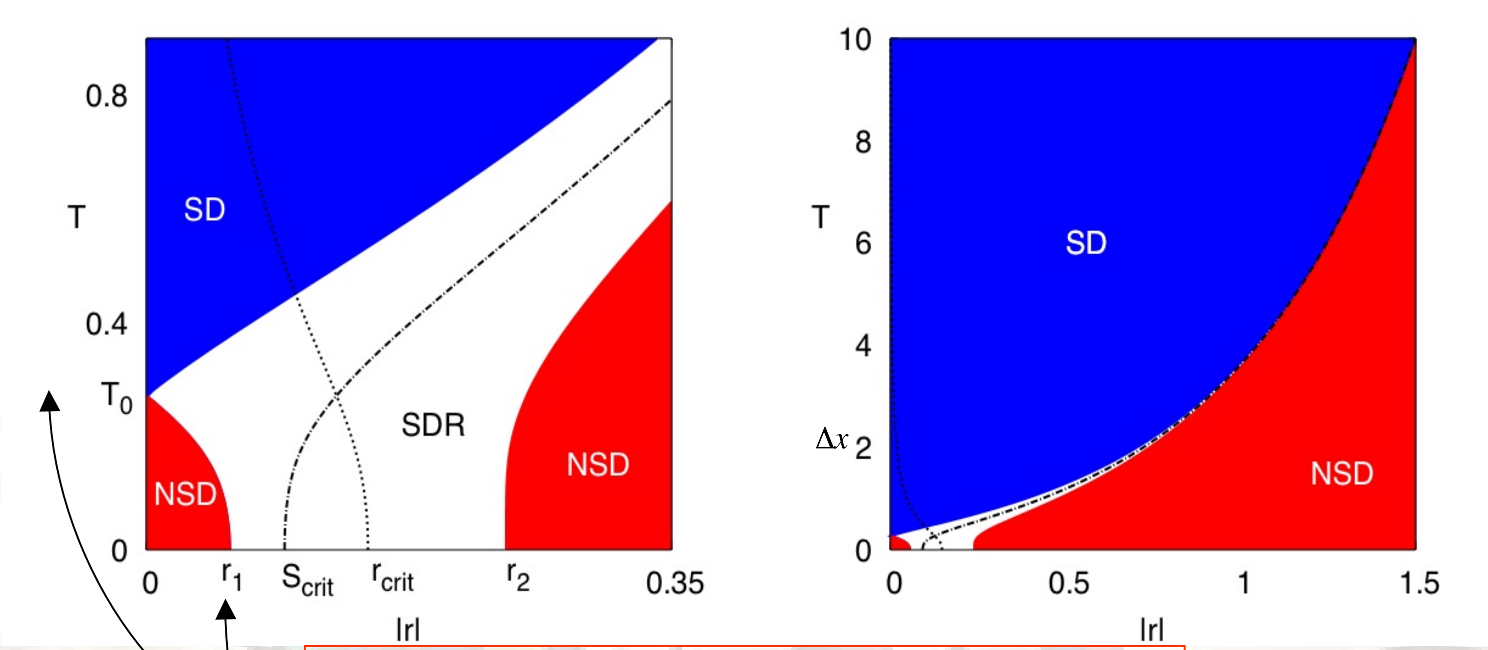
Analytical expressions can be obtained for the asymptotic values of the coefficients of the master equation for an *ohmic* environment. We can use them to construct a phase diagram



- Accurately describe all numerically simulations !

# THREE DYNAMICAL PHASES FOR ENTANGLEMENT

- Interesting points in the phase diagram :



$T_0$ is such that $\Delta x\big|_{T=T_0} = \sqrt{\frac{\hbar}{2m\Omega}}$

$r_1$ is such that $r_1 = \log\left(\frac{\sqrt{\frac{\hbar}{2m\Omega}}}{\Delta x(T=0)}\right)$

$$r_2 = \log\left(\frac{\Delta p(T=0)}{\sqrt{\frac{m\Omega\hbar}{2}}}\right)$$

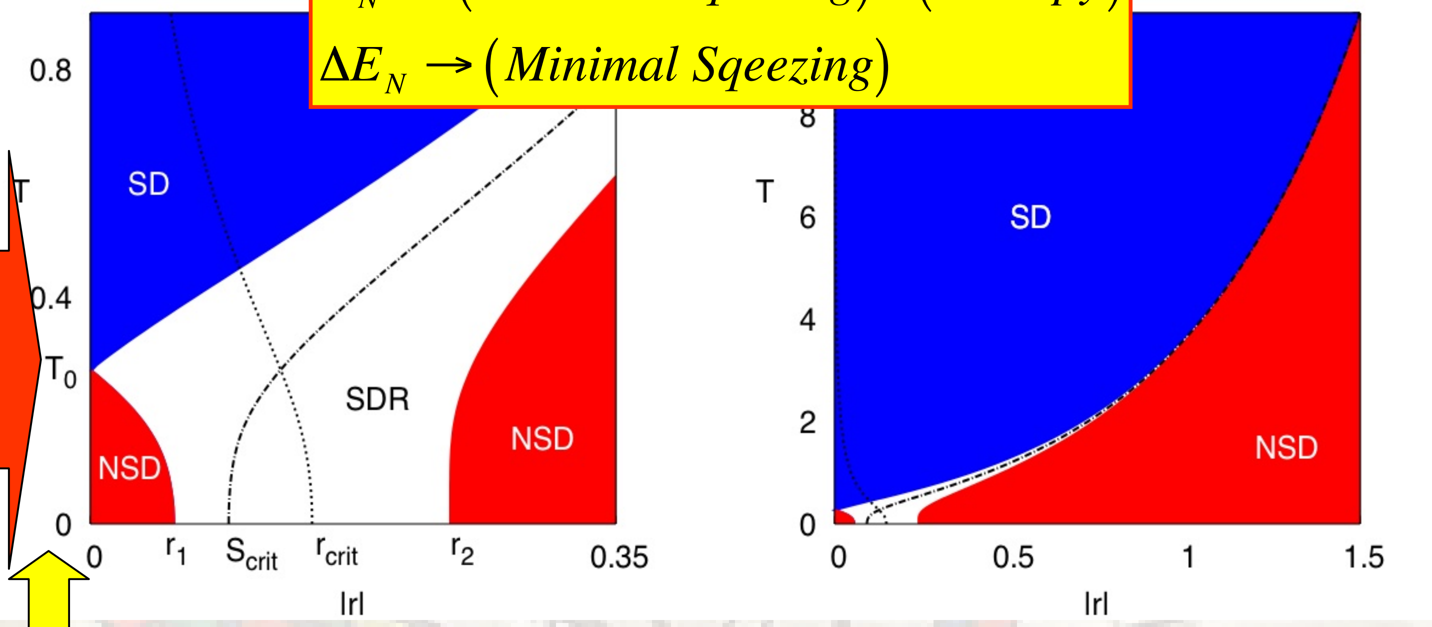
$$r_1 = \frac{1}{2}\log\left(\frac{\pi}{2}\,\frac{\sqrt{1-\gamma^2/\Omega^2}}{\arccos(\gamma/\Omega)}\right)$$

$$r_1 = \frac{1}{2}\log\left(\frac{1-4\gamma^2/\Omega^2}{\sqrt{1-\gamma^2/\Omega^2}}\arccos(\gamma/\Omega) + \frac{4}{\pi}\,\frac{\gamma}{\Omega}\log\left(\frac{\Lambda}{\Omega}\right)\right)$$

# THREE DYNAMICAL PHASES FOR ENTANGLEMENT

$$\tilde{E}_N \rightarrow (Maximal\ Sqeezing) - (Entropy)$$

$$\Delta E_N \rightarrow (Minimal\ Sqeezing)$$

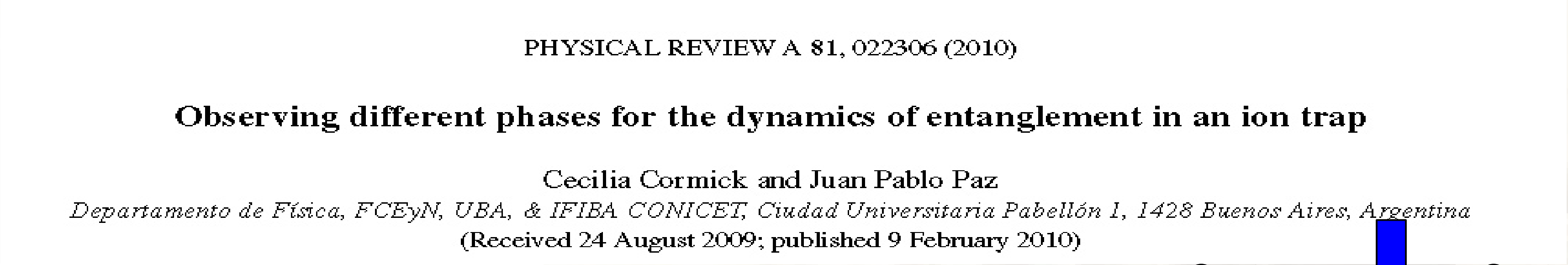


- NSD Island: Non Markovian, non perturbative
- Entanglement in the NSD Island COMES FROM the squeezing of the equilibrium state! (from the environment)

- SDR region (also non-Markovian) decreases width for ohmic environment as $\frac{1}{\Lambda}$

- SD region: entanglement dies because entropy is too large.
- NSD “continent”: entanglement persists because it is in a protected state.

# HOW TO OBSERVE THESE PHASES?



PHYSICAL REVIEW A 81, 022306 (2010)

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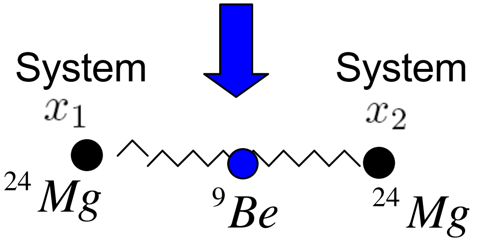
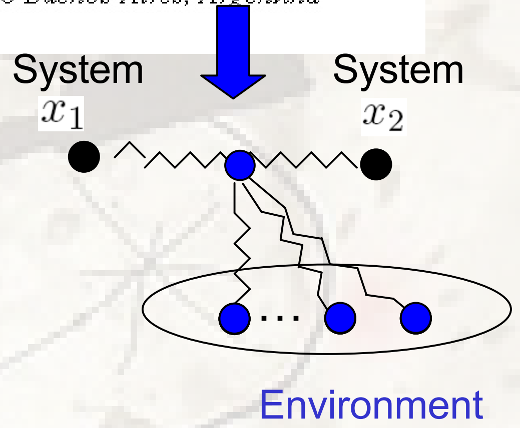


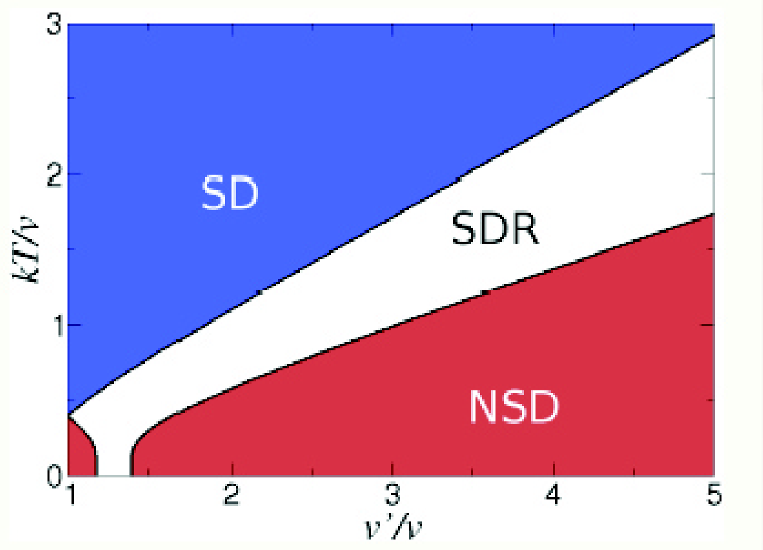


$$\omega_x = 1.68\,\omega_z \;\Rightarrow r_{crit} \approx 0.75, \quad S_{crit} \approx 0.32$$

$$\omega_x = 2.2\,\omega_z \;\Rightarrow r_{crit} \approx 0.47, \quad S_{crit} \approx 0.40$$

A ROUGH GUIDE THROUGH THE EXPERIMENT

- Three ions are cooled, y-potential is tight. Radial (x) modes are used for the simulation.
- Then central ion is cooled continuously. This cools two normal modes. “Temperature”: free parameter
- Squeezing of radial mode is created by varying x-trap frequency. Initial squeezing is free parameter.
- Final state dispersions are measured



# SUMMARY

**TIMESCALES FOR DECOHERENCE: SHORTEST IN THE MACROSCOPIC DOMAIN**

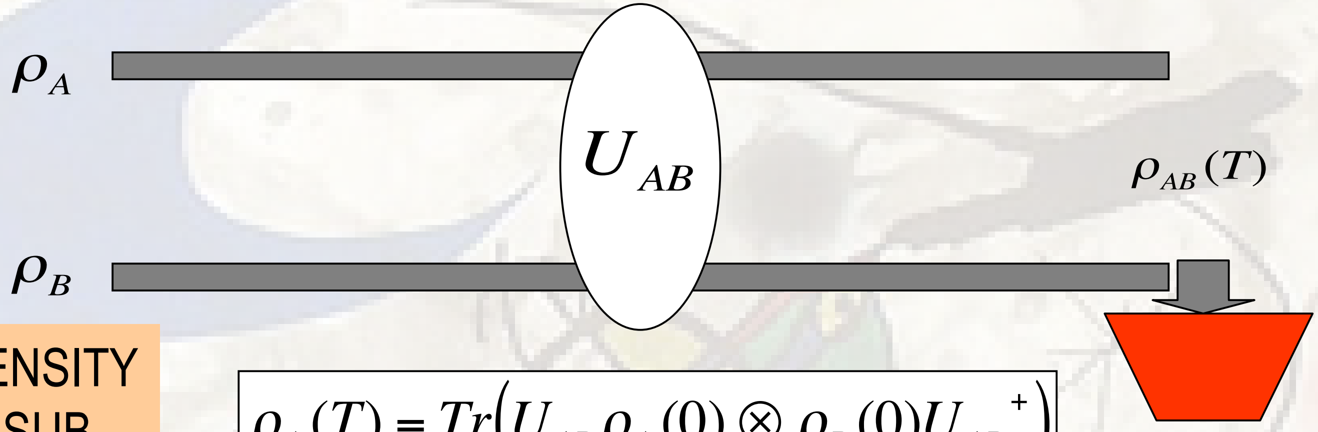
**PREFERRED STATES: PREDICTABILITY SIEVE**

**DECAY OF ENTANGLEMENT: NONTRIVIAL DYNAMICAL PHASES (EXPERIMENTALLY OBSERVABLE IN ION TRAPS?)**

**DECAY OF OTHER QUANTUM CORRELATIONS (DISCORD): N. FREITAS AND J.P.P., |Dynamics of Quantum Discord of Two Oscillators Coupled With the Same Environment", Phys. Rev. A85, 032118 (2012)**

# ANOTHER LOOK AT THE EVOLUTION OF QUANTUM OPEN SYSTEMS: INPUT-OUTPUT

MOST GENERAL EVOLUTION OF TWO INTERACTING SUBSYSTEMS



REDUCED DENSITY MATRIX OF SUB-SYSTEM

$$\rho_A(T) = Tr\left(U_{AB}\rho_A(0)\otimes\rho_B(0)U_{AB}{}^{+}\right)$$

$$\rho_B(0) = \left|\Psi_B(0)\right\rangle\left\langle\Psi_B(0)\right|;$$

$$A_b = \left\langle\phi_b\right|U_{AB}\left|\Psi_B(0)\right\rangle, \qquad \sum_b A_b{}^{+}A_b = I$$

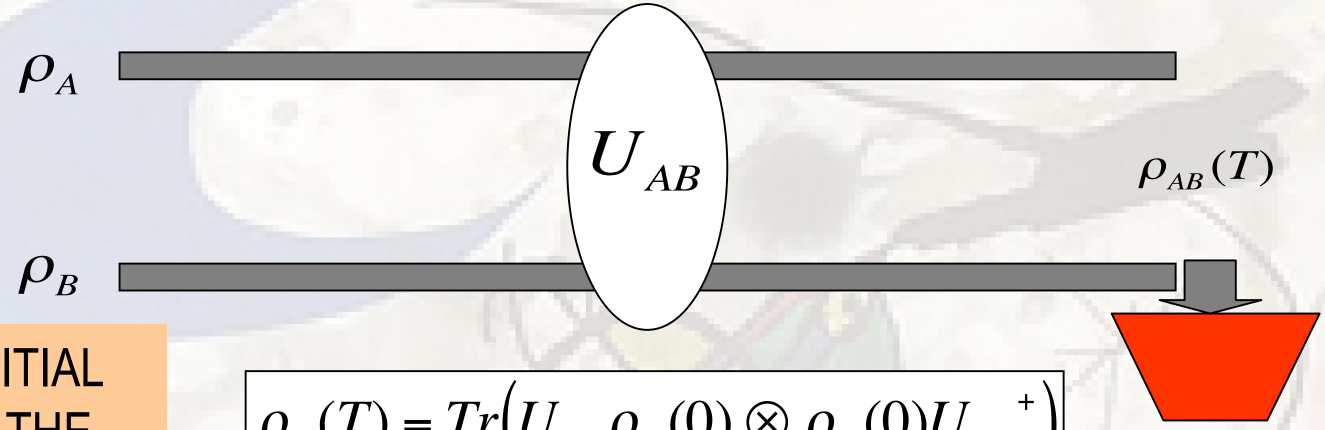
$$\rho_A(T) = \sum_b A_b\ \rho_A(0)\ A_b{}^{+}$$

KRAUSS REPRESENTATION

TIME EVOLUTION OF SUB-SYSTEM IS ENFORCED BY A LINEAR, TRACE PRESERVING, COMPLETELY POSITIVE MAP

# ANOTHER LOOK AT THE EVOLUTION OF QUANTUM OPEN SYSTEMS: INPUT-OUTPUT

HOW GENERAL IS THIS RESULT?



VALID IF INITIAL STATE OF THE ENVIRONMENT IS MIXED (OBVIOUS)

$$\rho_A(T) = Tr\left(U_{AB}\rho_A(0) \otimes \rho_B(0) U_{AB}{}^{+}\right)$$

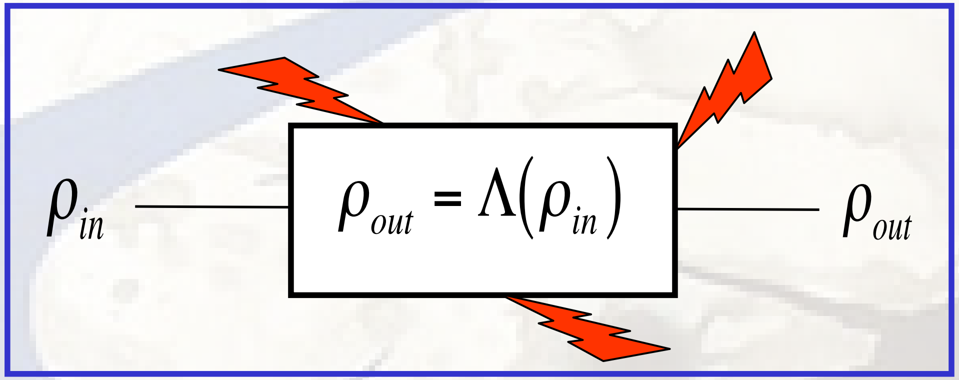
$$\rho_A(T) = \sum_b A_b\ \rho_A(0)\ A_b{}^{+}, \qquad \sum_b A_b{}^{+} A_b = I$$

KRAUSS FORM IF AND ONLY IF THE INITIAL STATE OF (A,B) HAS ZERO DISCORD (ONLY CLASSICAL CORRELATIONS); Lidar (2009)

$$\rho_A(0) = \sum_k p_k \rho_{A,k}(0) \otimes \rho_{B,k}(0)$$

$$Tr\left(\rho_{B,k}(0)\rho_{B,k'}(0)\right) = 0 \quad if \quad k \neq k'$$

# QUANTUM EVOLUTION AS A LINEAR MAP



A QUANTUM PROCESS IS A LINEAR MAP (PRESERVING HERMITICITY, TRACE AND POSITIVITY)

ANY LINEAR MAP IS DEFINED BY ITS ‘CHI-MATRIX’

$$\rho_{out} = \Lambda(\rho_{in}) = \sum_{ab} \chi_{ab} E_b \, \rho_{in} \, E_a^{+}$$

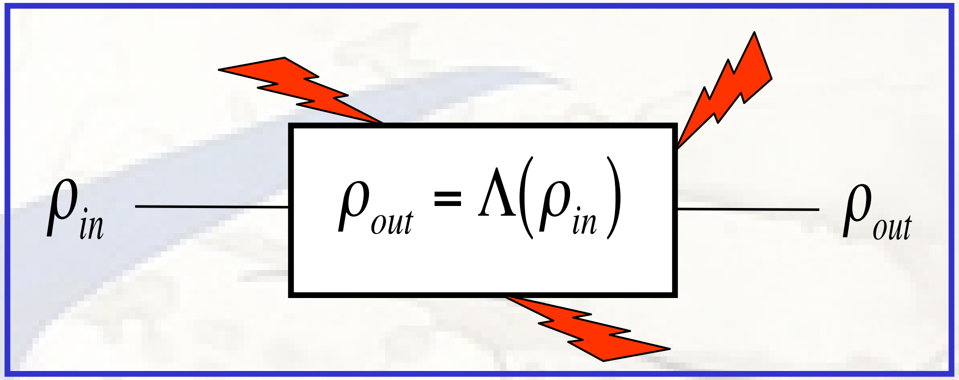
$$P_0 = I, \quad P_1 = X, \quad P_2 = Y, \quad P_3 = Z$$

$$E_a = P_{m_1} \otimes P_{m_2} \otimes \ldots\ldots\ldots \otimes P_{m_n}$$

$$E_0 = I_1 \otimes I_2 \otimes \ldots\ldots\ldots \otimes I_n$$

$$Tr(E_a E_b) = D\delta_{ab}$$

PROPERTIES OF THE MAP ARE IN ONE TO ONE CORRESPONDENCE WITH PROPERTIES OF ITS ‘CHI-MATRIX’



$$\rho_{out} = \Lambda(\rho_{in}) = \sum_{ab} \chi_{ab} E_b \, \rho_{in} \, E_a^{+}$$

- MAP IS HERMITIAN $\chi_{mn} = \chi_{nm}^{*}$
- MAP PRESERVES TRACE $\sum_{mn} \chi_{mn} E_m^{+} E_n = I$

$$\chi_{ab} = U_{ac} D_c U_{cb}^{+}$$

$$\Downarrow$$

$$\Lambda(\rho) = \sum_{a} D_c A_c \rho A_c^{+}$$

$$A_c = \sum_{b} U_{bc} E_b$$

**ANY LINEAR HERMITIAN MAP IS THE DIFFERENCE OF TWO CP MAPS**

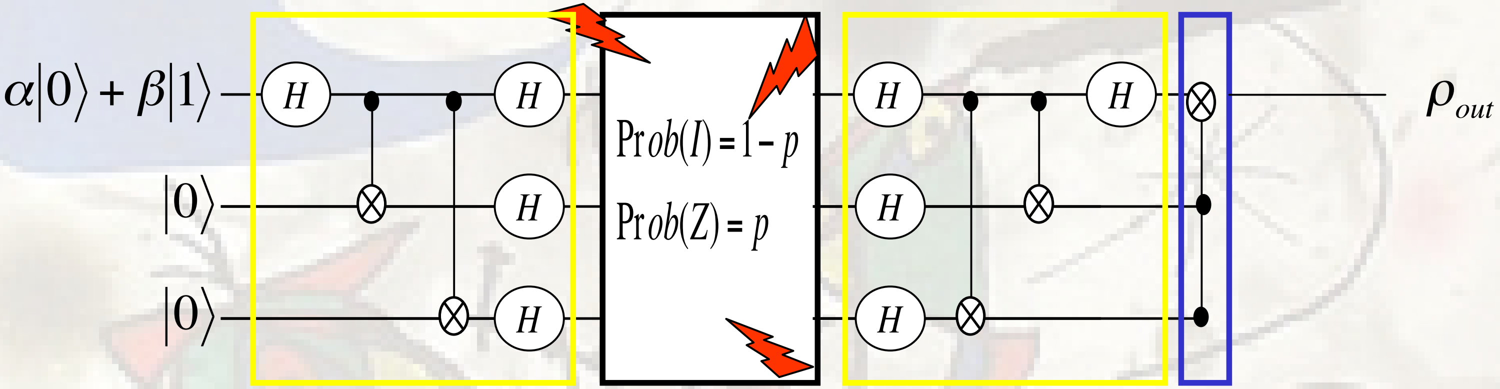
MAP $\Lambda(\rho)$ IS COMPLETELY POSITIVE (CP) $\Leftrightarrow$ MATRIX $\chi_{ab}$ IS POSITIVE

***KRAUS FORM OF THE MAP***

# THE NEED FOR QUANTUM PROCESS TOMOGRAPHY

IF YOU KNOW THE KRAUSS REPRESENTATION THEN YOU CAN DEVISE GOOD ERROR CORRECTION STRATEGIES

$$\rho_{out} = (1-p)\,\rho_{in} + p\,Z\rho_{in}Z$$



$$\rho_{out} = \rho_{in} + O(p^2), \quad F = Tr(\rho_{in}\rho_{in}) = 1 - O(p^2)$$

FIDELITY GOES FROM LINEAR TO QUADRATIC IN p!

BUT WHAT IF YOU DO NOT KNOW THE KRAUSS OPERATORS?

**NEED TO KNOW YOUR ENEMY TO BE ABLE TO DEFEAT HIM**

**QUANTUM PROCESS TOMOGRAPHY**

# WHY IS QUANTUM PROCESS TOMOGRAPHY HARD?

$$\rho_{out} = \Lambda\left(\rho_{in}\right) = \sum_{ab} \chi_{ab} E_b \, \rho_{in} \, E_a^{+}, \quad \sum_{ab} \chi_{ab} E_a^{+} E_b = I$$

- 1) THERE ARE EXPONENTIALLY MANY COEFFICIENTS $\chi_{mn}$

(i.e. There are $D^2 \times D^2$ of them where $D = 2^n$)

- 2) TO FIND OUT ANY ONE OF THEM WE NEED EXPONENTIAL RESOURCES

***STANDARD QUANTUM PROCESS TOMOGRAPHY (SQPT)***

***Chapter 10, Nielsen & Chuang's book***

$$P_{ik} = Tr\left(\rho_k \Lambda\left(\rho_i\right)\right)$$

- EXPERIMENTALLY DETERMINE “TRANSITION PROBABILITIES”

$$P_{ik} = \sum_{nm} \chi_{mn} Tr\left(\rho_k E_n \, \rho_i E_m^{+}\right)$$

- FIND $\chi_{mn}$ INVERTING (HUGE) LINEAR SYSTEM.

“STANDARD QUANTUM PROCESS TOMOGRAPHY”
(NIELSEN & CHUANG, CHAPTER 10)

# QUANTUM PROCESS TOMOGRAPHY IS HARD

## LECTURE 5

**ARE THERE GOOD (EFFICIENT) METHODS FOR QUANTUM PROCESS TOMOGRAPHY?**

**YES (FOR PARTIAL QPT, EFFICIENT METHODS TO EXTRACT USEFUL INFORMATION)**

# QUANTUM OPEN SYSTEMS, DECOHERENCE AND QUANTUM PROCESS TOMOGRAPHY

JUAN PABLO PAZ
Quantum Foundations and Information @ Buenos Aires
QUFIBA: *http://www.qufiba.df.uba.ar*
*Departamento de Fisica Juan José Giambiagi, FCEyN, UBA, Argentina*

APLICATIONS OF QUANTUM MECHANICS
CUERNAVACA, MEXICO
JULY 2012

Departamento de Física Juan José Giambiagi

Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires

# PLAN

**Lecture 1: Decoherence and the quantum origin of the classical world. Evolution of quantum open systems. Quantum Brownian motion as a paradigm. Master equation.**

**Lecture 2: General results on Dynamics and Thermodynamics of linear quantum open systems. Emergence of the laws of thermodynamics. E.A. MARTINEZ & J.P.P. arXiv:12074256**

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**Lecture 5: New methods for Quantum Process Tomography. “Selective and efficient QPT”. Theory and experimental implementation with single photons.**

# This talk

UM PROCESS TOMOGRAPHY

essary? Why is it hard? Standar

nd Efficient Quantum Process To

os Aires

HYSICAL REVIEW LETTERS

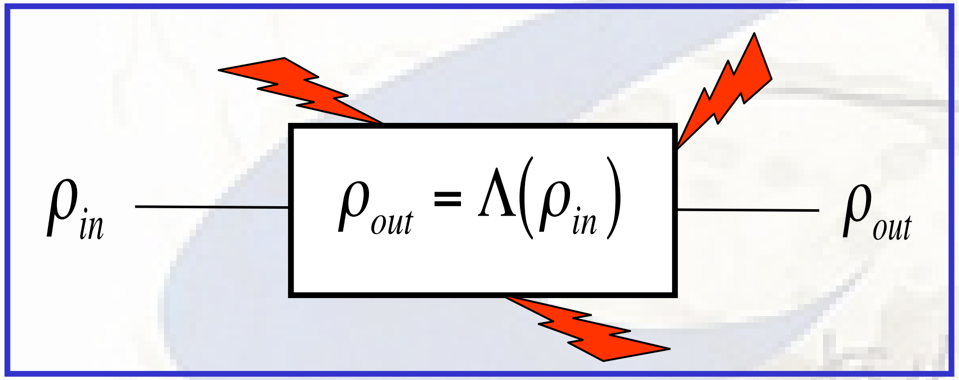
week ending
2 SEPTEMBER 2011

## Selective and Efficient Quantum Process Tomography without Ancilla

Christian Tomás Schmiegelow,[1] Ariel Bendersky,[1] Miguel Antonio Larotonda,[2] and Juan Pablo Paz[1]
[1]*Departamento de Física & IFIBA, FCEyN, UBA, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
[2]*CEILAP, CITEDEF, J.B. de La Salle 4397, 1603 Villa Martelli, Buenos Aires, Argentina*
(Received 10 February 2011; published 1 September 2011)

**An improved quantum algorithm for process tomography AND its experimental implementation on a 2-qubit system (2 qubits in 1 photon)**

# WHAT IS QUANTUM PROCESS TOMOGRAPHY?



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ANY LINEAR MAP IS DEFINED BY ITS 'CHI-MATRIX'

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$$E_m = P_{m_1} \otimes P_{m_2} \otimes \ldots\ldots\ldots \otimes P_{m_n}$$

$$E_0 = I_1 \otimes I_2 \otimes \ldots\ldots\ldots \otimes I_n$$

$$Tr(E_m E_n) = D\delta_{mn}$$

- MAP IS HERMITIAN $\chi_{mn} = \chi_{nm}^{*}$
- MAP PRESERVES TRACE $\sum_{mn} \chi_{mn} E_m^{+} E_n = I$

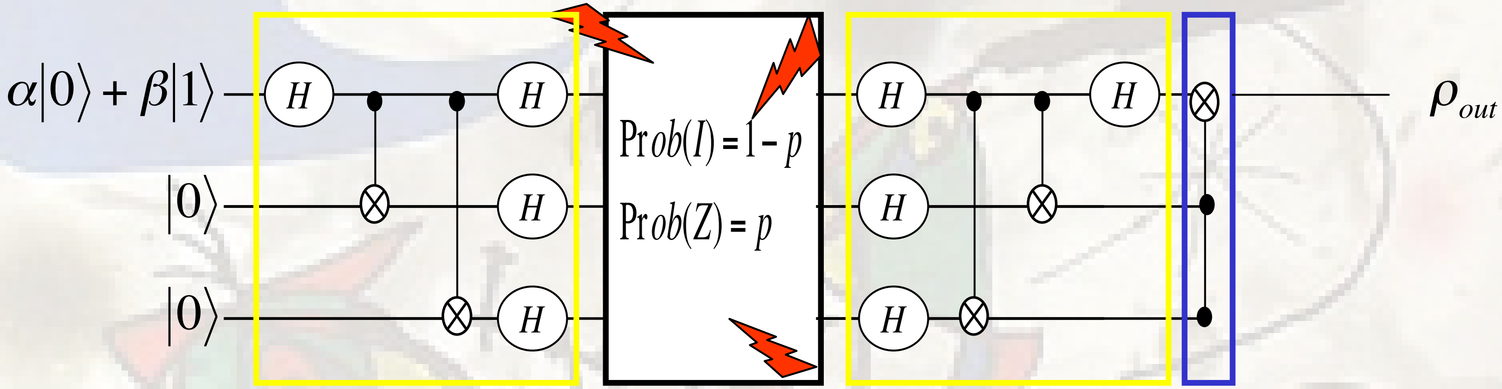
MAP $\Lambda(\rho)$ IS COMPLETELY POSITIVE (CP) ⟺ MATRIX $\chi_{mn}$ IS POSITIVE

# IMPORTANCE OF QUANTUM PROCESS TOMOGRAPHY

a) FUNDAMENTAL

b) NECESSARY TO AVOID DECOHERENCE

$$\rho_{out} = (1-p)\,\rho_{in} + p\, Z\rho_{in}Z$$



$$\rho_{out} = \rho_{in} + O(p^2), \quad F = Tr(\rho_{in}\rho_{in}) = 1 - O(p^2)$$

ERRORS MAY BE CORRECTED (fidelity goes from linear to quadratic in p)

**BUT TO DO THAT IT IS NECESSARY TO CHARACTERIZE THE TYPICAL ERRORS AND FOR THIS WE NEED QUANTUM PROCESS TOMOGRAPHY (QPT)**

# WHY IS QUANTUM PROCESS TOMOGRAPHY HARD?

$$\rho_{out} = \Lambda(\rho_{in}) = \sum_{nm} \chi_{mn} E_n \, \rho_{in} \, E_m^{+}, \quad \sum_{mn} \chi_{mn} E_m^{+} E_n = I$$

- 1) THERE ARE EXPONENTIALLY MANY COEFFICIENTS $\chi_{mn}$

(i.e. There are $D^2 \times D^2$ of them where $D = 2^n$)

- 2) TO FIND OUT ANY ONE OF THEM WE NEED EXPONENTIAL RESOURCES

***STANDARD QUANTUM PROCESS TOMOGRAPHY (SQPT)***

***Chapter 10, Nielsen & Chuang's book***

$$P_{ik} = Tr\left(\rho_k \Lambda(\rho_i)\right)$$

- EXPERIMENTALLY DETERMINE "TRANSITION PROBABILITIES"

$$P_{ik} = \sum_{nm} \chi_{mn} Tr\left(\rho_k E_n \, \rho_i E_m^{+}\right)$$

- FIND $\chi_{mn}$ INVERTING (HUGE) LINEAR SYSTEM.

"STANDARD QUANTUM PROCESS TOMOGRAPHY"
(NIELSEN & CHUANG, CHAPTER 10)

# QUANTUM PROCESS TOMOGRAPHY IS HARD!

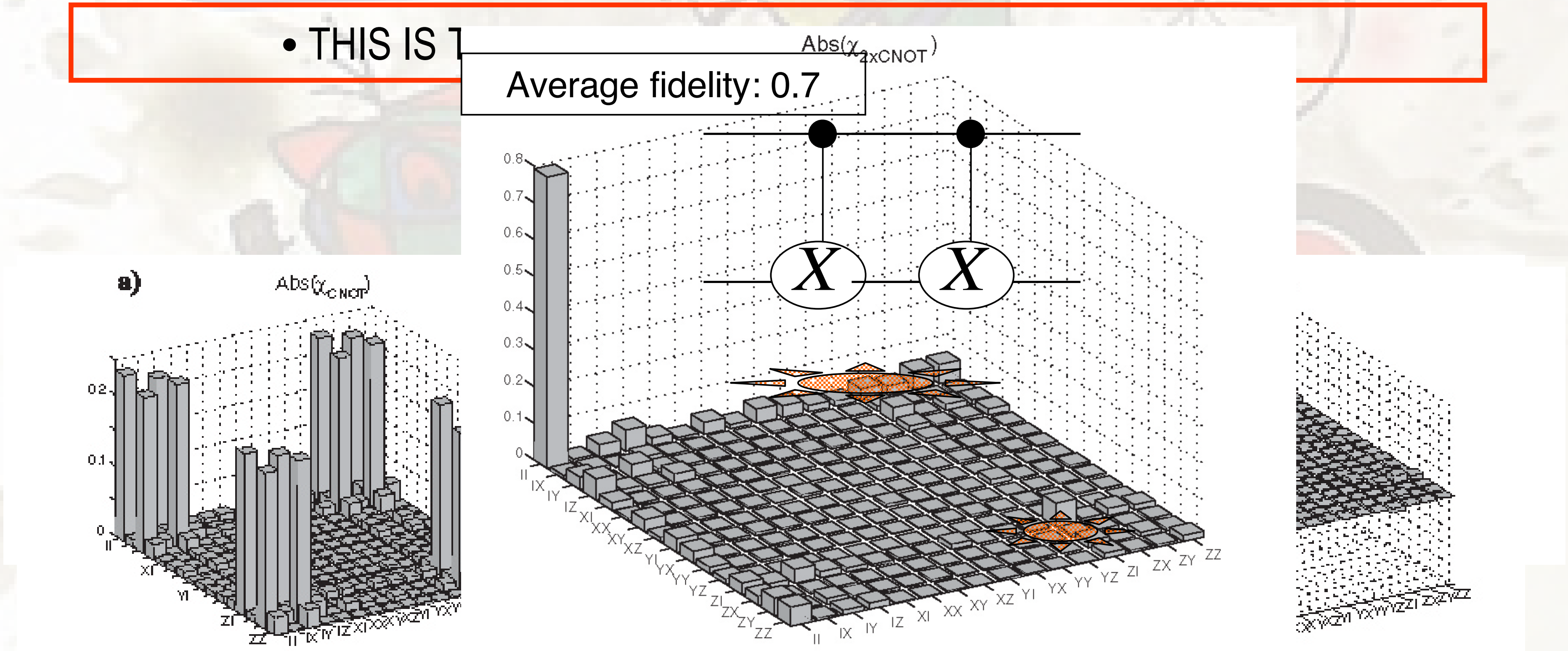
***QUANTUM RESOURCES*** (qbits, operations, state preparations, measurements, etc):

- prepare $D^2$ initial states and detect $D^2$ final states
- repeat experiments (exponentially) many times to geta fixed precision in $\chi_{mn}$

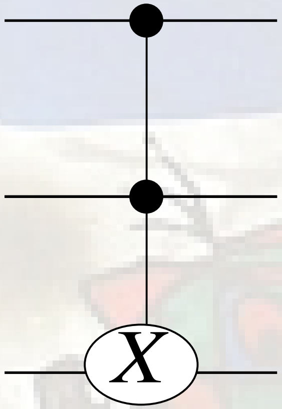
***CLASSICAL RESOURCES*** (data processing in classical computers):

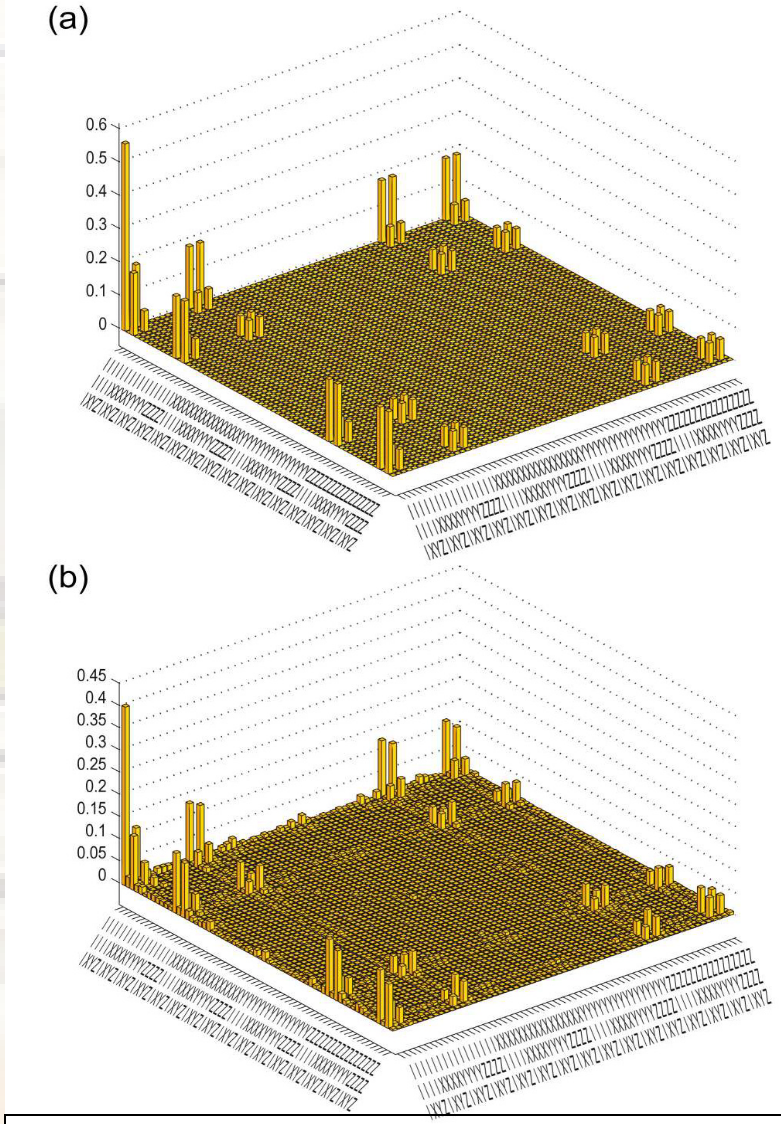
- invert an exponentially large (linear) system

- THIS IS T


Average fidelity: 0.7

Realization of the quantum Tofoli gate with trapped ions" T Monz, K. Kim, W. Hänsel, M. Riebe, A. S. Villar, P. Schindler, M. Chwalla, M. Hennrich, and R. Blatt, Physical Review Letters 102, 040501 (2009)





64x64 matrix. Obtained after inverting a 4096x4096 linear system formed with all the probabilities measured after perfirnubg 4096 experiments (prepare each of 64 independent states and measure each of 64 independent transition probabilities.

Average fidelity: 0.67
Measured Chi-matrix shows the same "fingerprint" of the ideal one (Tofoli)

# THIS TALK: AN ALTERNATIVE APPROACH FOR QPT

- 1) SELECT A COEFFICIENT $\chi_{mn}$ (OR A SET OF THEM)
- 2) DIRECTLY MEASURE THEM WITHOUT DOING FULL QUANTUM PROCESS TOMOGRAPHY

***QUANTUM AND CLASSICAL RESOURCES*** Poly(Log(D))

***METHOD BASED ON A PROPERTY OF THE CHI-MATRIX***

***MATRIX ELEMENTS ARE AVERAGED SURVIVAL PROBABILITIES OF A CHANNEL***

$$F_{mn}(\Lambda) = \int d|\Psi\rangle\,\langle\Psi|\,\Lambda\big(E_m|\Psi\rangle\langle\Psi|E_n\big)\,|\Psi\rangle$$

$$F_{mn}(\Lambda) = \frac{1}{(D+1)}\left(D\chi_{mn} + \delta_{mn}\right)$$

Estimate $F_{mn}(\Lambda)$ → Estimate $\chi_{mn}$

- A simple consequence of the following identity

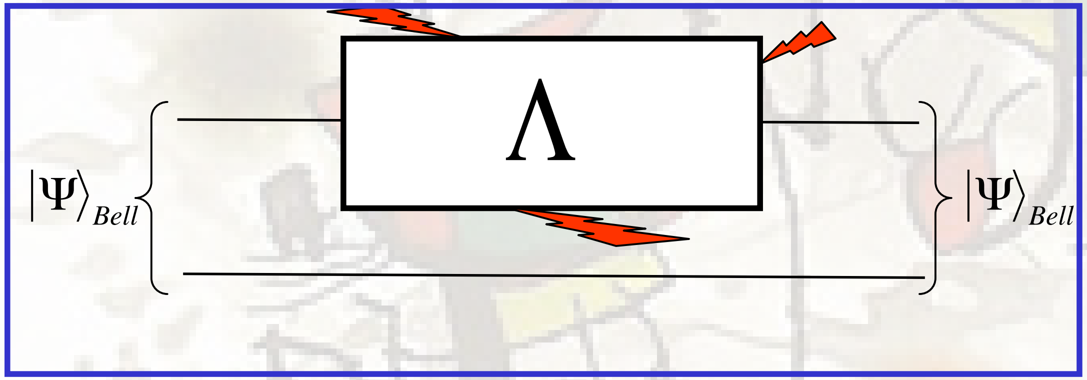
$$\int d|\Psi\rangle\,\langle\Psi|\,A\,|\Psi\rangle\,\langle\Psi|\,B|\Psi\rangle = \frac{1}{D(D+1)}\big(Tr(AB) + Tr(A)Tr(B)\big)$$

# QPT IS HARD… (continuation)

- STANDARD QUANTUM PROCESS (NIELSEN & CHUANG) IS EXPONENTIALLY HARD EVEN TO ACHIEVE PARTIAL CHARACTERIZATION!!

ARE THERE OTHER METHODS? DCQP (‘DIRECT CARACTERIZATION OF A QUANTUM PROCESS). D. Lidar and M. Mohseni, Phys. Rev. A 77, 032322 (2008)

- DIAGONAL MATRIX ELEMENTS $\chi_{nn}$ ARE SURVIVAL PROBABILITIES OF SYSTEM PLUS ANCILLA (A VERY EXPENSIVE RESOURCE!)



$$|\Psi\rangle_{Bell} = \frac{1}{\sqrt{D}}\sum_{j=1}^{D}|j\rangle\otimes|j\rangle$$

$$\chi_{00} = {}_{Bell}\langle\Psi|\Lambda\otimes I\left(|\Psi\rangle_{Bell}\langle\Psi|_{Bell}\right)|\Psi\rangle_{Bell}$$

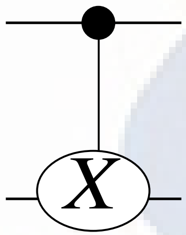
- BUT OFF DIAGONAL ELEMENTS ARE STILL EXPONENTIALLY HARD TO CALCULATE

- NEED AN EXPENSIVE RESOURCE: CLEAN ANCILLA THAT INTERACTS WITH OUR SYSTEM

# THE IMPORTANCE OF BEING SELECTIVE…

QUESTION: How close are we approaching a “target” operation?

$$U_T^+ \rho U_T = \sum_{nm} \chi_{mn}^{(T)} E_n \, \rho \, E_m^{\;+}$$



Example: C-NOT (chi-matrix has only 16 non-vanishing elements)

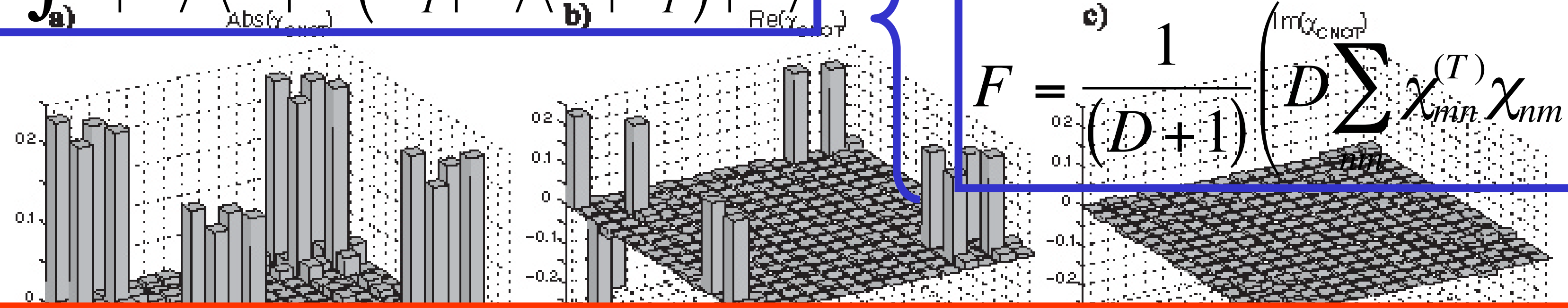
$$U_T = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X = \frac{1}{2}(I+Z)\otimes I + \frac{1}{2}(I-Z)\otimes X$$

AVERAGE FIDELITY PROVIDES A GOOD WAY TO QUANTIFY THIS

$$F = \int d|\Psi\rangle \, \langle\Psi| \, \Lambda\left(U_T^+ |\Psi\rangle\langle\Psi| U_T\right) |\Psi\rangle$$

$$F = \sum_{nm} \chi_{mn}^{(T)} F_{nm}$$

$$F = \frac{1}{(D+1)}\left(D\sum_{nm} \chi_{mn}^{(T)} \chi_{nm} + 1\right)$$



MORAL: DON’T NEED THE FULL MATRIX TO COMPUTE F (ONLY 16 ELEMENTS!, INDEPENDENT OF D!!)

# TWO PROBLEMS MAKE NEW METHOD LOOK IMPOSSIBLE!

$$F_{mn}(\Lambda) = \int d|\Psi\rangle \langle\Psi| \, \Lambda\left(E_m|\Psi\rangle\langle\Psi|E_n\right)|\Psi\rangle$$

$$F_{mn}(\Lambda) = \frac{1}{(D+1)}\left(D\chi_{mn} + \delta_{mn}\right)$$

PERFORMING $\int d|\Psi\rangle$

IN THE LABORATORY???

IMPLEMENTING $E_m|\Psi\rangle\langle\Psi|E_n$

IN THE LABORATORY???

**THIS TALK: TWO SOLUTIONS!**

1) **How to perform the integral over the entire Hilbert space?**
2) **How to apply a non physical (non CP) map?**

# HOW TO INTEGRATE IN HILBERT SPACE?

- USE 2-DESIGNS!
- A SET OF STATES (S) IS A 2-DESIGN IF AND ONLY IF

$$\int d|\Psi\rangle \langle\Psi| A |\Psi\rangle \langle\Psi| B|\Psi\rangle = \frac{1}{\#(S)} \sum_{|\Phi_J\rangle \in S} \langle\Phi_j| A|\Phi_j\rangle \langle\Phi_j| B|\Phi_j\rangle$$

- 2-designs are powerful tools!! USEFUL RESULTS

a) 2-DESIGNS EXIST!

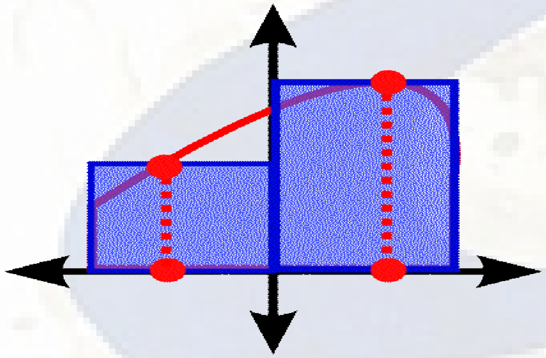
b) THEY HAVE AT LEAST $D^2$ STATES

c) STATES OF (D+1) MUTUALLY UNBIASED BASIS FORM A 2-DESIGN

d) EFFICIENT ALGORITHMS TO GENERATE 2-DESIGNS EXIST

# INTERLUDE ON 2-DESIGNS

- IS THE EXISTENCE OF 2-DESIGN A SURPRISE?

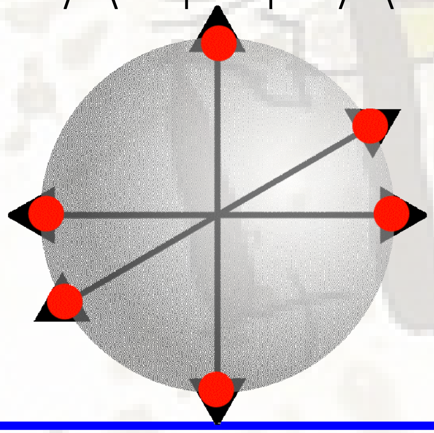


$$\int_0^1 dx\; f(x) = \int_0^1 dx\left(a+bx+cx^2\right) = \frac{1}{2}\left(f(x_1)+f(x_2)\right)$$

$$x_{\substack{1\\2}} = \frac{1}{2} \pm \sqrt{\frac{1}{12}} \Rightarrow S = \{x_1,x_2\} \to 2 - design$$

- 2-DESIGNS FOR SPIN 1/2: ENABLE TO COMPUTE AVERAGES OF PRODUCTS OF TWO EXPECTATION VALUES (integrals of functions that depend upon TWO bras and TWO kets)

$$\begin{aligned}\int d|\Psi\rangle\,\langle\Psi|\,A\,|\Psi\rangle\,\langle\Psi|\,B|\Psi\rangle &= \frac{1}{6}\langle\uparrow_x|A|\uparrow_x\rangle\langle\uparrow_x|B|\uparrow_x\rangle + \frac{1}{6}\langle\downarrow_x|A|\downarrow_x\rangle\langle\downarrow_x|B|\downarrow_x\rangle\\ &+\frac{1}{6}\langle\uparrow_y|A|\uparrow_y\rangle\langle\uparrow_y|B|\uparrow_y\rangle + \frac{1}{6}\langle\downarrow_y|A|\downarrow_y\rangle\langle\downarrow_y|B|\downarrow_y\rangle\\ &+\frac{1}{6}\langle\uparrow_z|A|\uparrow_z\rangle\langle\uparrow_z|B|\uparrow_z\rangle + \frac{1}{6}\langle\downarrow_z|A|\downarrow_z\rangle\langle\downarrow_z|B|\downarrow_z\rangle\end{aligned}$$



# HOW TO APPLY A NON CP MAP?

TRANSFORM IT INTO THE DIFFERENCE BETWEEN CP MAPS…

$$F_{mn}(\Lambda)=\frac{1}{D(D+1)}\sum_j \langle\Phi_j|\,\Lambda\big(E_m|\Phi_j\rangle\langle\Phi_j|E_n\big)|\Phi_j\rangle$$

$$F^{\pm}{}_{mn}(\Lambda)=\frac{1}{D(D+1)}\sum_j \langle\Phi_j|\,\Lambda\Big(\underbrace{(E_m\pm E_n)|\Phi_j\rangle}\langle\Phi_j|(E_m\pm E_n)\Big)|\Phi_j\rangle$$

$$\mathrm{Re}\big(F_{mn}(\Lambda)\big)=\frac{1}{2}\big(F^{+}{}_{mn}(\Lambda)+F^{-}{}_{mn}(\Lambda)\big)$$

CAN WE PREPARE THOSE STATES?

OPERATOR BASI $E_m$

2-DESIGN $|\Phi_j\rangle$

SPLIT THE OPERATOR BASE INTO (D+1) COMMUTING SUBSETS

(each set contains the identity and D-1 commuting operators)

EACH COMMUTING SET OF $E_m$ OPERATORS DEFINES A BASIS

ALL SUCH (D+1) BASES ARE MUTUALLY UNBIASEDrs)

# HOW TO APPLY A NON CP MAP?

USE 2-DESIGN DEFINED BY THE (D+1) MUBs ASSOCIATED WITH THE SPLIT OF THE OPERATOR BASIS $E_m$

$$\left|\Phi_j\right\rangle \rightarrow \left|\Phi^{(b)}{}_k\right\rangle; b = 1,\ldots,D+1; \quad k = 1,\ldots,D$$

$$E_m\left|\Phi^{(b)}{}_k\right\rangle \approx \left|\Phi^{(b)}{}_{k'}\right\rangle$$

$$\left(E_m \pm E_n\right)\left|\Phi^{(b)}{}_k\right\rangle \approx \left(\left|\Phi^{(b)}{}_{k'}\right\rangle \pm \left|\Phi^{(b)}{}_{k''}\right\rangle\right)$$

EFFICIENT PROCCEDURE FOR PREPARING SUCH STATES EXIST!

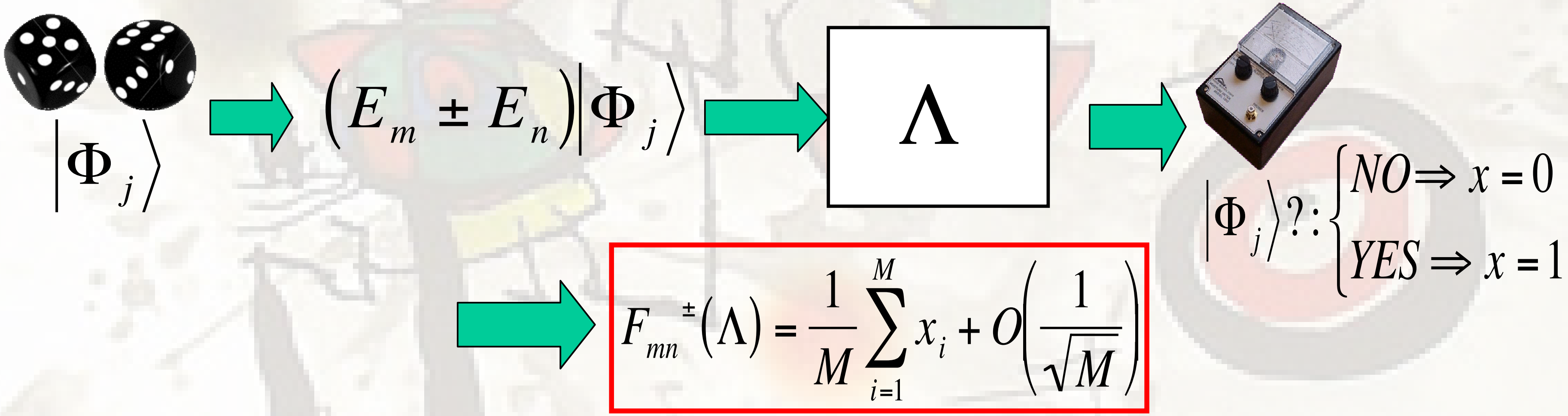
# NEW METHOD: SELECTIVE AND EFFICIENT Q.P.T.

FIRST EFFICIENT METHOD TO DETERMINE ANY ELEMENT OF CHI MATRIX OF A QUANTUM PROCESS

Poly(Log(D)) QUANTUM GATES REQUIRED

Poly(Log(D)) CLASSICAL POST-PROCESSING REQUIRED

NO ANCILLARY RESOURCES (CLEAN QUBITS) ARE REQUIRED



$$|\Phi_j\rangle \rightarrow (E_m \pm E_n)|\Phi_j\rangle \rightarrow \boxed{\Lambda} \rightarrow$$

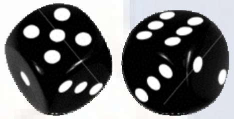
$$|\Phi_j\rangle ?: \begin{cases} NO \Rightarrow x = 0 \\ YES \Rightarrow x = 1 \end{cases}$$

$$\rightarrow \boxed{F_{mn}^{\pm}(\Lambda) = \frac{1}{M}\sum_{i=1}^{M} x_i + O\left(\frac{1}{\sqrt{M}}\right)}$$

# PHOTONIC IMPLEMENTATION: (EXPERIMENT IN OUR LAB IN BUENOS AIRES)

FULLY CHARACTERIZING A QUANTUM CHANNEL AFFECTING TWO QUBITS ENCODED IN A SINGLE (HERALDED) PHOTON

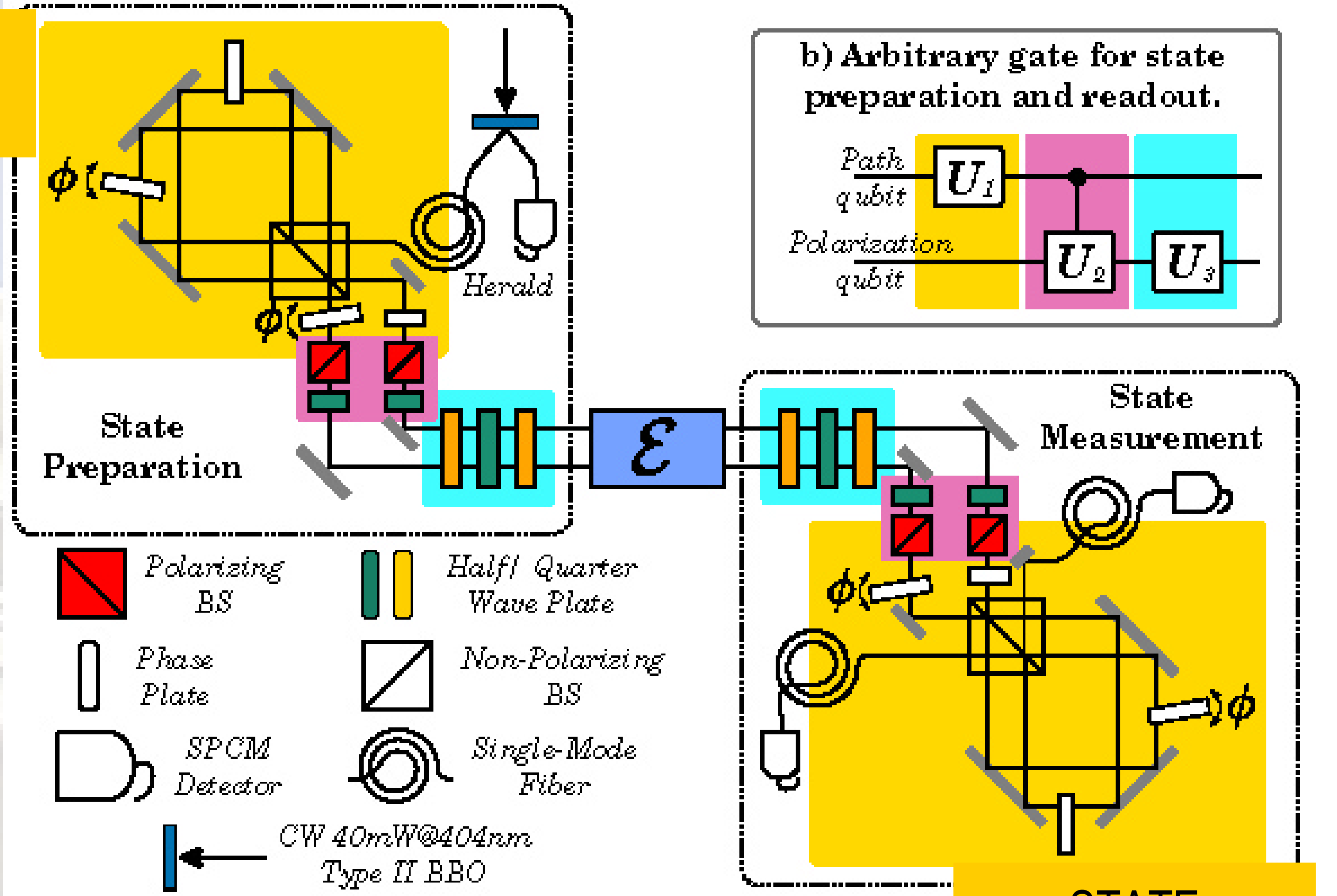
STATE PREPARATION



$$|\Phi_j\rangle$$

⬇

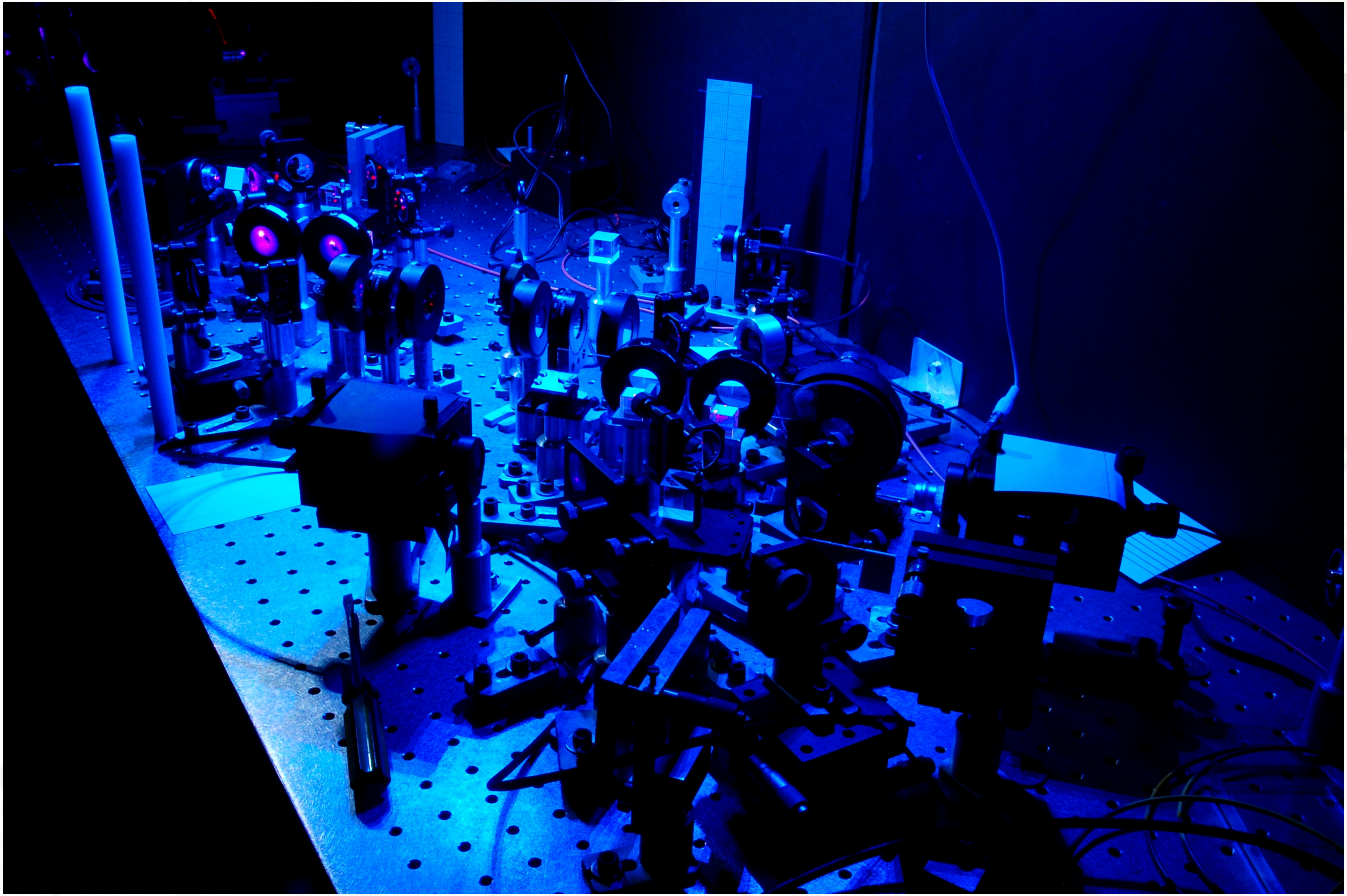
$$(E_m \pm E_n)|\Phi_j\rangle$$



STATE DETECTION

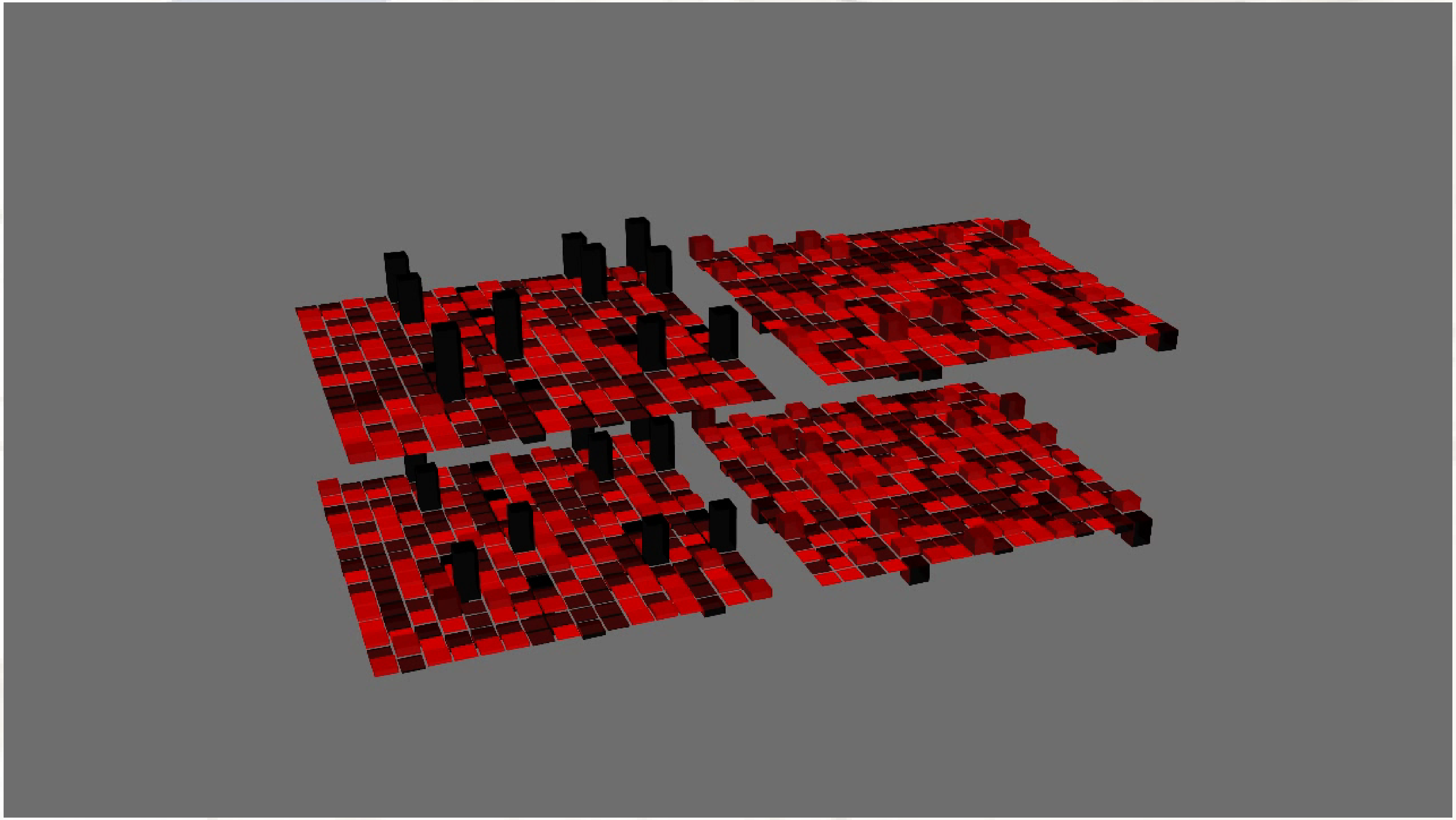
$$|\Phi_j\rangle$$

# PHOTONIC IMPLEMENTATION (EXPERIMENT IN OUR LAB IN BUENOS AIRES)



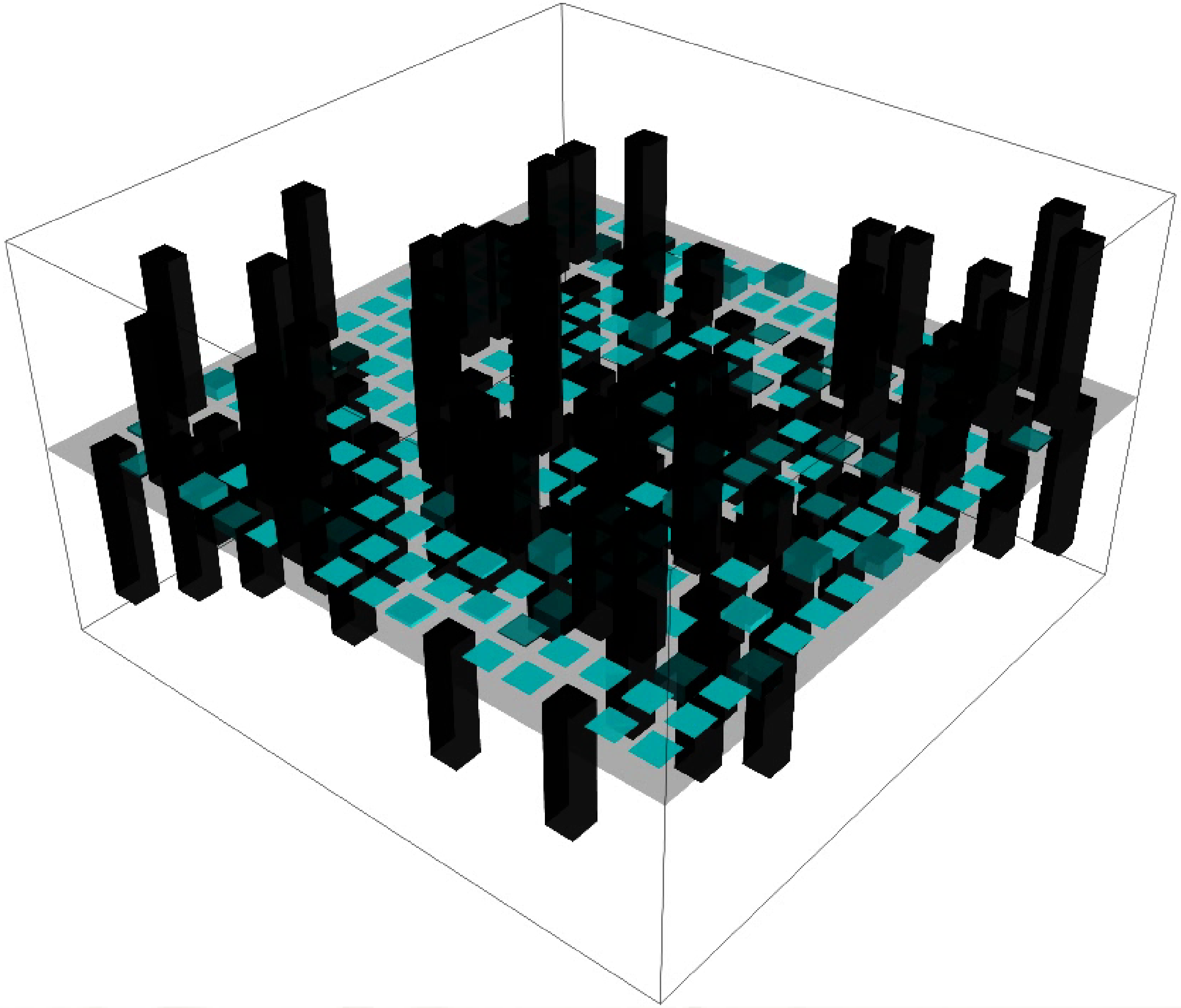
# FULL QUANTUM PROCESS TOMOGRAPHY

OUR METHOD GIVES PERFECT AGREEMENT WITH STANDARD QPT (alla NIELSEN AND CHUANG)



# FULL QUANTUM PROCESS TOMOGRAPHY

IMPORTANT: EACH MATRIX ELEMENT IS ESTIMATED BY SAMPLING OVER THE 2-DESIGN. PRECISION INCREASES WITH SAMPLE SIZE



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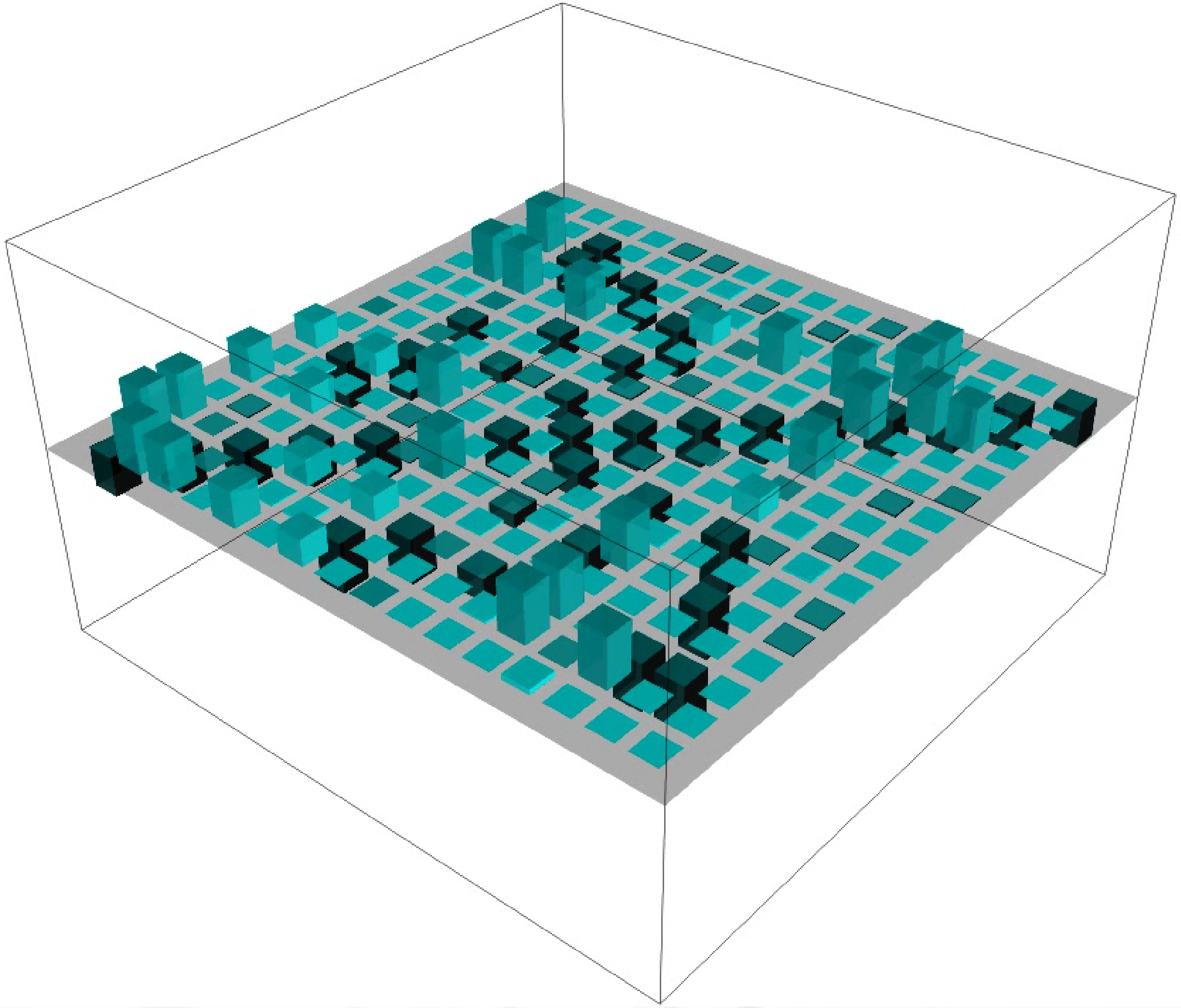
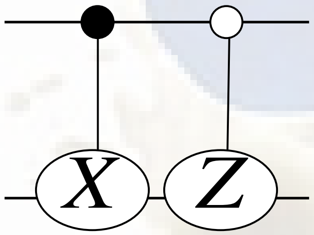


# POWER OF THE METHOD: SAMPLING & PARTIAL QPT

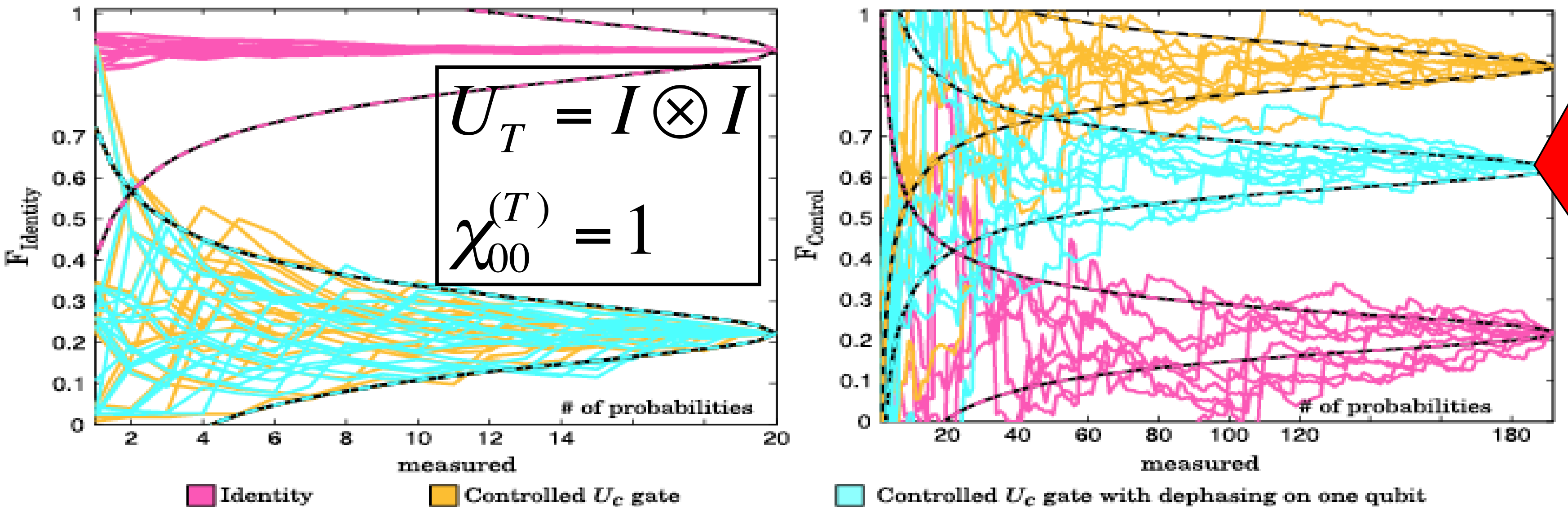
COMPUTE FIDELITY OF A QUANTUM GATE WITHOUT DOING FULL QUANTUM PROCESS TOMOGRAPHY!

$$U_T^+ \rho U_T = \sum_{nm} \chi_{mn}^{(T)} E_n \, \rho \, E_m^{\;+}$$

$$F = \int d|\Psi\rangle \, \langle\Psi| \, \Lambda\left(U_T^+|\Psi\rangle\langle\Psi|U_T\right) |\Psi\rangle = \sum_{nm} \chi_{mn}^{(T)} F_{nm}$$



$$U_T = |0\rangle\langle 0| \otimes Z + |1\rangle\langle 1| \otimes X = \frac{1}{2}(I+Z) \otimes Z + \frac{1}{2}(I-Z) \otimes X$$



# THE IMPORTANCE OF BEING SELECTIVE…

QUESTION: How close are we approaching a “target” operation?

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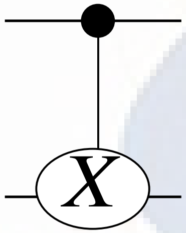

Example: C-NOT (chi-matrix has only 16 non-vanishing elements)

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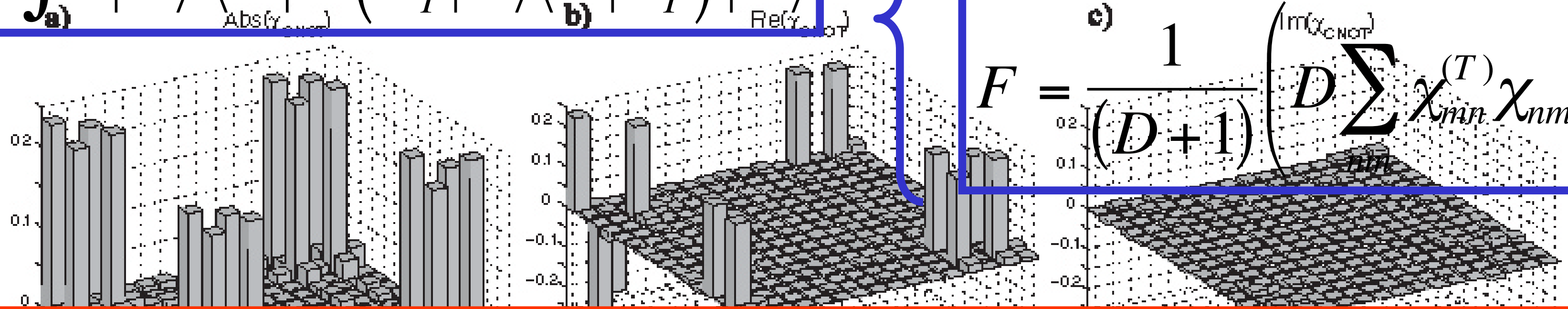
$$F = \sum_{nm} \chi_{mn}^{(T)} F_{nm}$$

$$F = \frac{1}{(D+1)}\left(D\sum_{nm} \chi_{mn}^{(T)} \chi_{nm} + 1\right)$$

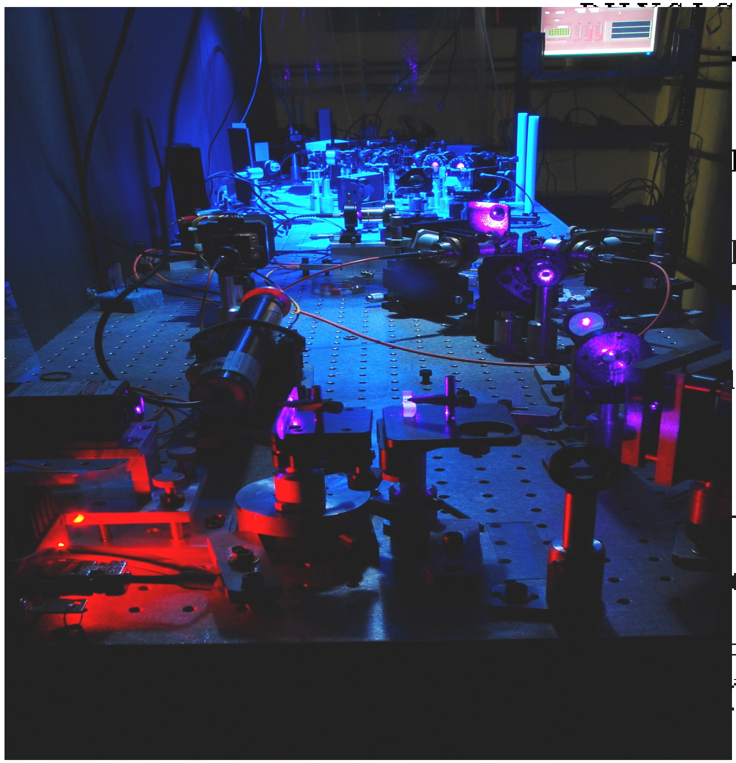


MORAL: DON’T NEED THE FULL MATRIX TO COMPUTE F (ONLY 16 ELEMENTS!, INDEPENDENT OF D!!)

# SUMMARY

FULL QPT IS ALWAYS HARD. STANDARD METHODS FOR PARTIAL QPT ARE ALSO EXPONENTIALLY HARD

- THERE IS AN ALTERNATIVE METHOD FOR EFFICIENT AND SELECTIVE PARTIAL QUANTUM PROCESS TOMOGRAPHY
- IT INVOLVES ESTIMATION OF ‘SURVIVAL PROBABILITIES’ OF A SET OF STATES FORMING A 2-DESIGN (VERY USEFUL RESOURCE!)



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