Two bodies gravitational system with variable mass and damping-anti damping effect due to star wind

Gustavo López Velázquez

Ma. Esmeralda Juárez

Departamento de Física, CUCEI

Universidad de Guadalajara

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OUTLINE

Comets as an example of variable mass system.

Two body problem and damping-anti damping model.

One degree of freedom equation of motion.

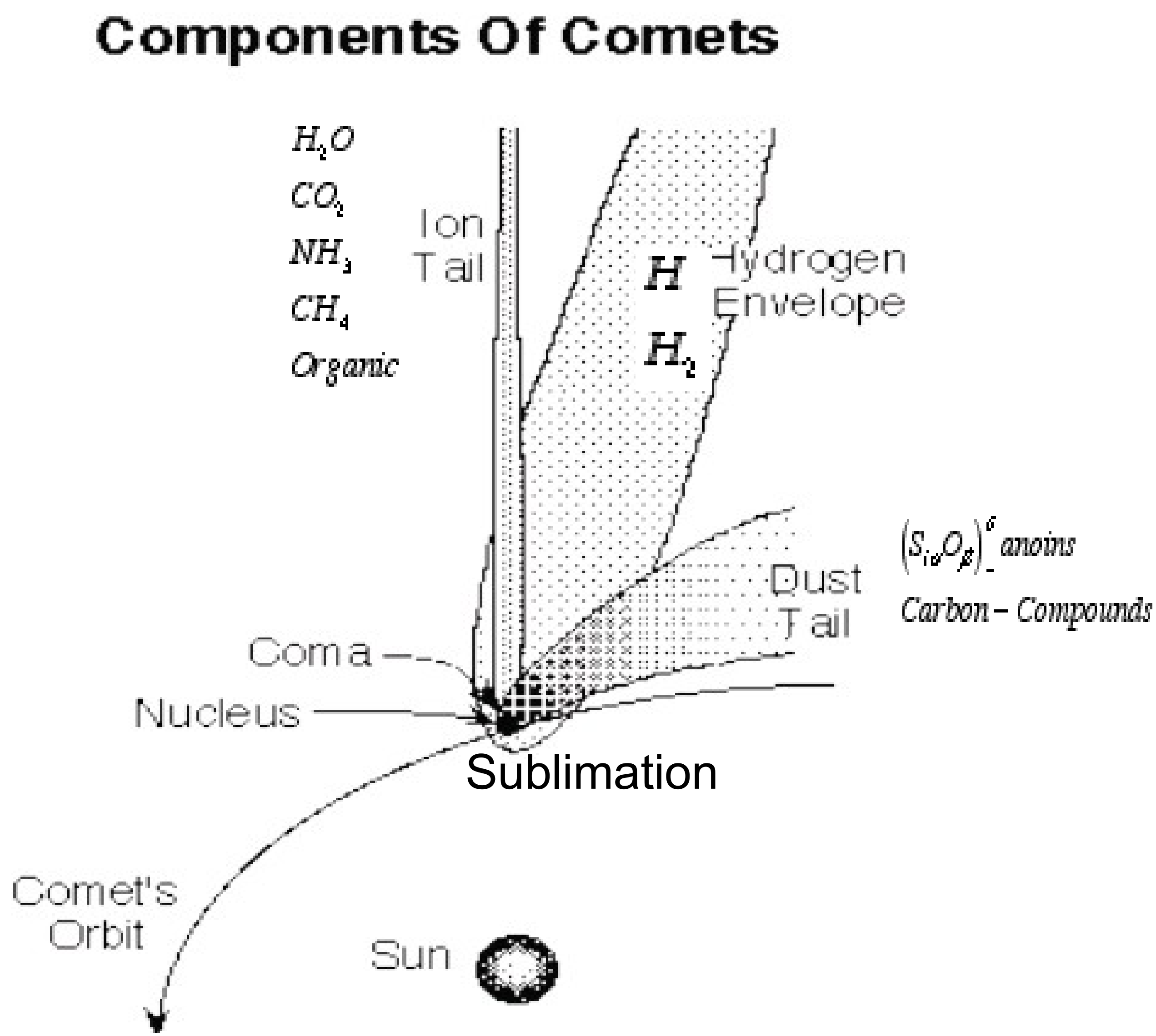
Radial constant of motion, Lagrangian, and Hamiltonian.

Model for variable mass.

Spiral behavior of the comet and period of motion of motion.

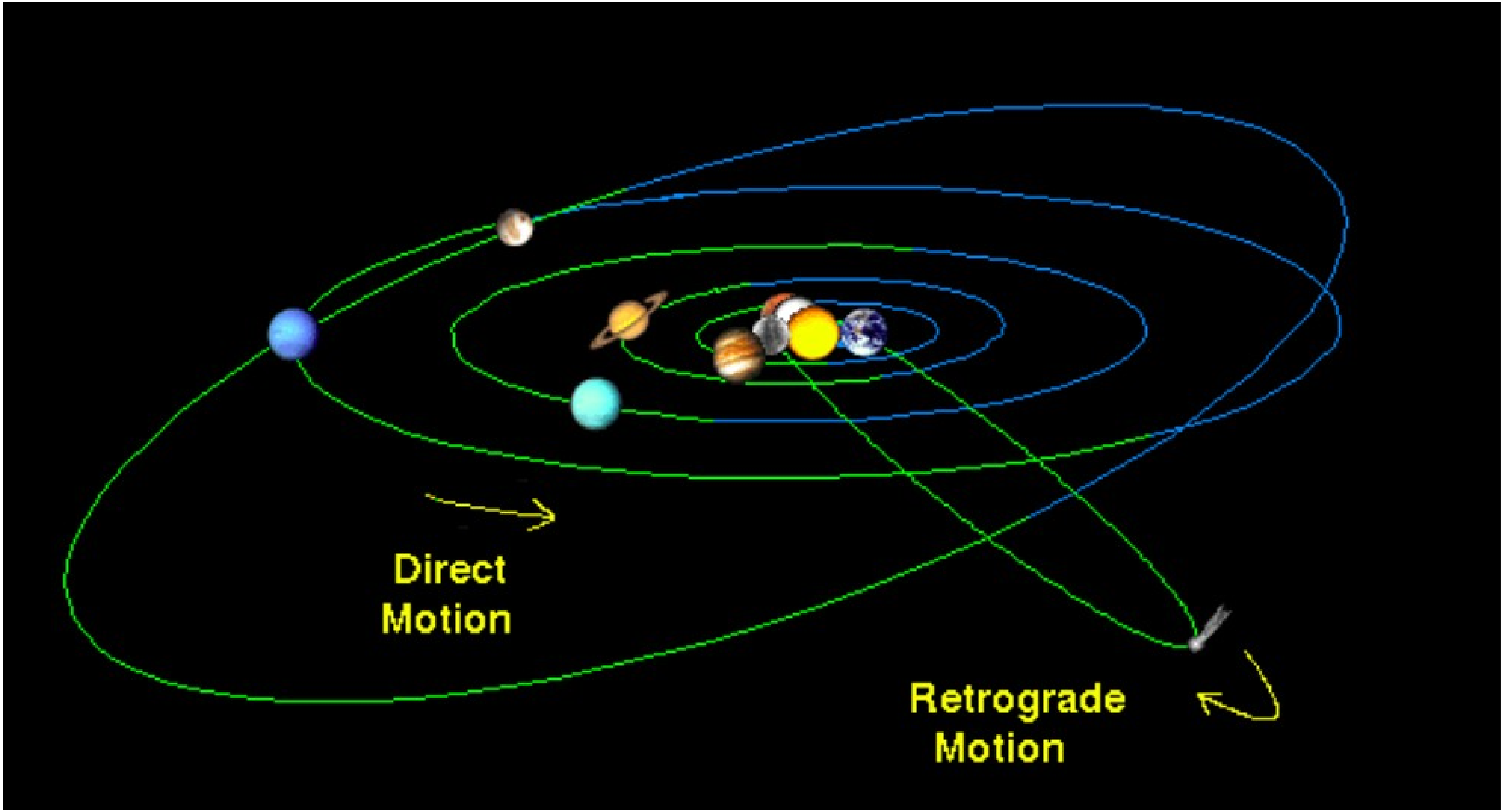
Conclusion.

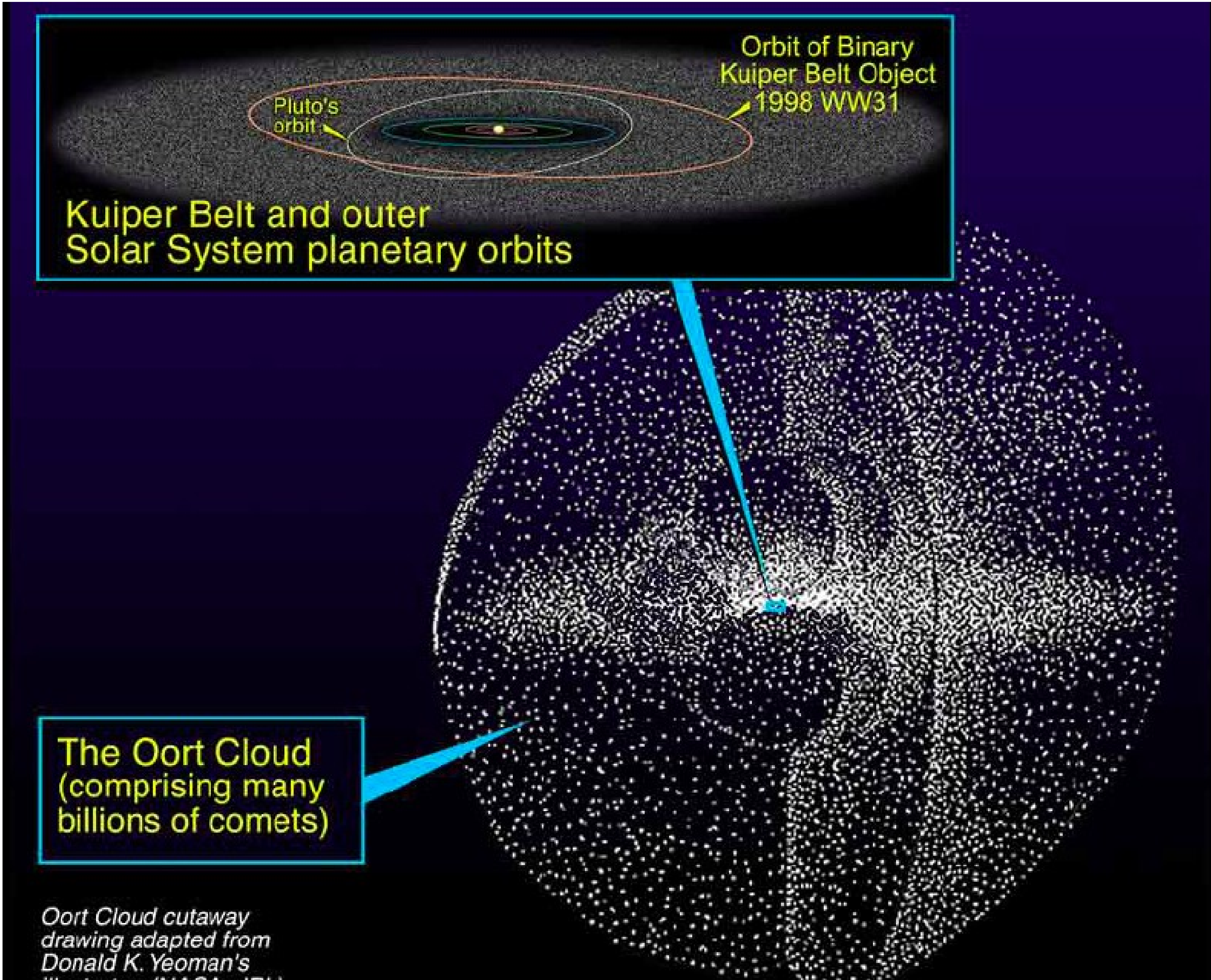
Comet


Components Of Comets
H_2O
CO_2
NH_3
CH_4
Organic
Ion Tail
H
H_2
Hydrogen Envelope
anoins
Carbon – Compounds
Dust Tail
Coma
Nucleus
Sublimation
Comet's Orbit
Sun


The tail of the comet always points away from the sun

Halley Comet





Oort Cloud cutaway
drawing adapted from
Donald K. Yeoman's

Equation of Motion

Meshcherski-Gylden (1900) problem
L.M. Berkovich, Celestial Mech. 24 (1981) 407
A.A. Bekov, Astr. Zh. 66 (1989) 135
C Prieto J.A.Docobo, Ast. Ast. 318 (1997) 657

$$\frac{d}{dt}\left(m_1 \frac{d\vec{r}_1}{dt}\right) = -\frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

$$\frac{d}{dt}\left(m_2 \frac{d\vec{r}_2}{dt}\right) = -\frac{Gm_1m_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) - \frac{\gamma}{|\vec{r}_1 - \vec{r}_2|}\left(\frac{d\,|\vec{r}_1 - \vec{r}_2|}{dt}\right)^2(\vec{r}_1 - \vec{r}_2)$$

c.m. coordinate system does not help $\longrightarrow$ Reference system at $\vec{r}_1 = \vec{0}$

$$\vec{r}_2 = \vec{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)$$

$$m_2(r - r\dot{\theta}^2 - r\dot{\varphi}\sin^2\theta) = -\frac{Gm_1m_2}{r^2} - m_2 r - \gamma r^2$$

$$m_2(2\dot{r}\dot{\theta} + r\dot{\theta} + r\dot{\varphi}\sin\theta\cos\theta) = -m_2 r\dot{\theta}$$

$$m_2(2r\dot{\varphi}\sin\theta + r\dot{\varphi}\sin\theta + 2r\dot{\varphi}\dot{\theta}\cos\theta) = -m_2 r\dot{\varphi}\sin\theta$$

$$\dot{\varphi} = 0$$

$$m_2(r - r\dot{\theta}^2) = -\frac{Gm_1m_2}{r^2} - m_2 r - \gamma r^2$$

$$m_2(2r\dot{\theta} + r\dot{\theta}) + m_2 r\dot{\theta} = 0. \longrightarrow l_\theta = m_2 r^2\dot{\theta}$$

$$\ddot{r} = -\frac{Gm_1}{r^2} - \frac{\dot{m}_2}{m_2}\dot{r} - \frac{\gamma}{m_2}\dot{r}^2 + \frac{l_\theta^2}{m_2^2 r^3}$$

$$m_2 = m_2(r) \Longrightarrow \dot{m}_2 = m'_2 \dot{r} \qquad m' = \frac{dm}{dr}$$

$$\ddot{r} = -\frac{Gm_1}{r^2} - \frac{m'_2 + \gamma}{m_2}\dot{r}^2 + \frac{l_\theta^2}{m_2^2 r^3} \Longrightarrow \begin{cases} \dot{r} = v \\ \dot{v} = -\dfrac{Gm_1}{r^2} - \dfrac{m'_2 + \gamma}{m_2} v^2 + \dfrac{l_\theta^2}{m_2^2 r^3} \end{cases}$$

$$v\frac{\partial K}{\partial r} + \left[-\frac{Gm_1}{r^2} - \frac{m'_2 + \gamma}{m_2} v^2 + \frac{l_\theta^2}{m_2^2 r^3}\right]\frac{\partial K}{\partial v} = 0 \qquad K(r,v) = G(C(x,v))$$

$$C(r,v) = m_2^2 e^{2\gamma\Lambda(r)} v^2 + \int dr\left(\frac{2Gm_1}{r^2} - \frac{2l_2^2}{m_2^2 r^3}\right) m_2^2 e^{2\gamma\Lambda(r)}$$ (characteristic curve)

$$\Lambda(r) = \int \frac{dr}{m_2(r)}$$

By choosing G as: $G_\pm(C) = \dfrac{C^\pm}{2m_{20}^\pm}$

$$K^{\pm}(r,v)=\frac{\left(m_2^{\pm}(r)\right)^2 v^2}{2m_{20}^{\pm}}e^{2\gamma/c\,t(v)}+V_{eff}^{\pm}(r)$$

$$K^{\pm}\xrightarrow[\overline{m_2=cte}]{\gamma\to 0}E=\frac{1}{2}m_{20}v^2+V_{eff}(r)$$

$$V_{eff}^{\pm}(r)=\frac{Gm_1}{m_{20}^{\pm}}\int\frac{\left(m_2^{\pm}(r)\right)^2 e^{2\gamma/c\,t(v)}dr}{r^2}-\frac{l_\theta^2}{m_{20}^{\pm}}\int\frac{e^{2\gamma/c\,t(v)}dr}{r^3}$$

$$V_{eff}(r)=-\frac{Gm_1m_{20}}{r}+\frac{l_\theta^2}{2m_{20}r^2}$$

It has an minimum at r* $\left(\frac{d^2V_{eff}^{\pm}}{dr^2}\right)_{r=r^*}>0$

$$r^*\left(m_2(r^*)\right)^2=\frac{l_\theta^2}{Gm_1}$$

G.Darboux, (1986)
E.T. Whittaker, (1917)
J.Douglas, Amer. Math. Soc.,50 (1941) 71.
J.A. Kobussen, Acta Phys Austriaca, 51, (1979) 293.
C. Leubner, Physica A, 86 (1981) 2.
G.López, Ann. Phys. 251 (1996) 383.
G. López, Int. J. Theo. Phys. (2008)

$$L^{\pm}=v^2\int\frac{K^{\pm}(r,v)dv}{v^2}\qquad p^{\pm}=\frac{\partial L^{\pm}}{\partial v}\qquad H^{\pm}=vp^{\pm}-L^{\pm}$$

$$L^{\pm}=\frac{\left(m_2^{\pm}(r)\right)^2 v^2}{2m_{20}^{\pm}}e^{2\gamma/c\,t(v)}-V_{eff}^{\pm}(r)$$

$$p=\frac{\left(m_2^{\pm}(r)\right)^2 e^{2\gamma/c\,t(v)}}{m_{20}^{\pm}}v$$

$$H^{\pm}=\frac{m_{20}^{\pm}p^2}{2\left(m_2^{\pm}(r)\right)^2}e^{-2\gamma/c\,t(v)}+V_{eff}^{\pm}(r)$$

$$v = (\pm) e^{-\gamma\Lambda^{\pm}(r)} \sqrt{\frac{2m_{20}^{\pm}}{\left(m_2^{\pm}(r)\right)^2}} \left[K_0^{\pm} - V_{eff}^{\pm}(r)\right]^{1/2}$$

$$p = (\pm) e^{+\gamma\Lambda^{\pm}(r)} \sqrt{\frac{2\left(m_2^{\pm}(r)\right)^2 m_{20}^{\pm}}{m_{20}^{\pm}}} \left[H_0^{\pm} - V_{eff}^{\pm}(r)\right]^{1/2}$$

Mass lost model

$$m_2^{\pm}(r) = \begin{cases} m_2^{-}(r_0)\left(1 - e^{-\alpha r}\right) \ldots\ldots\ldots\ldots\ldots (+) v < 0 \\ m_2^{+}(r_p) - b\left(1 - e^{-\alpha(r - r_p)}\right) \ldots\ldots (-) v > 0 \end{cases}$$

$$\alpha = \alpha(\delta m)$$

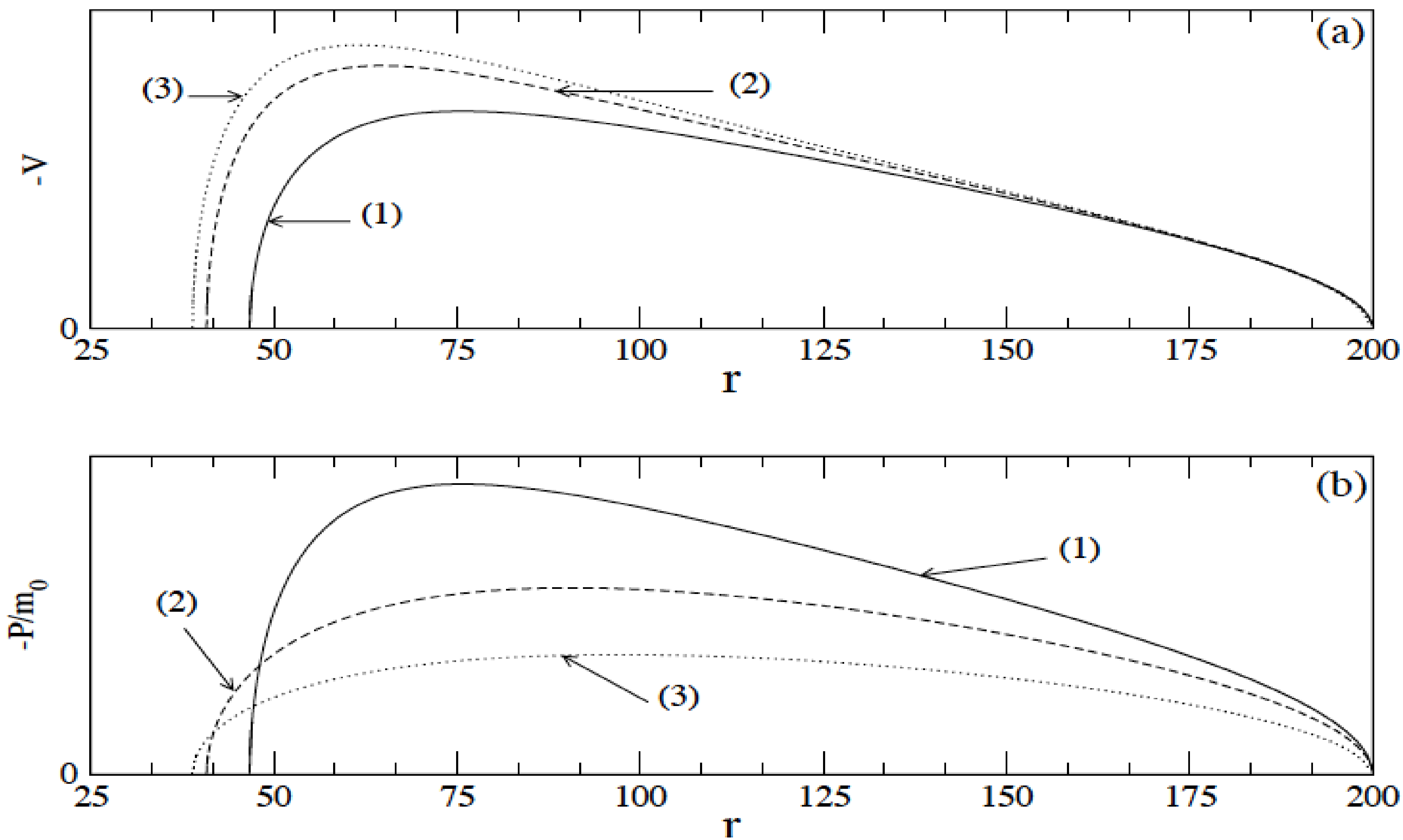
$$b = b(\delta m)$$

$$\Lambda^{+}(r) = \frac{1}{\alpha m_0} \ln\left(e^{\alpha r} - 1\right)$$

$$\Lambda^{-}(r) = \frac{-1}{\alpha(b - m_p)} \left[\alpha r + \ln\left(m_p - b(1 - e^{-\alpha(r - r_p)})\right)\right]$$

$$V_{eff}^{+}(r) = \left[-\frac{G m_1 m_2}{r} + \frac{l_\theta^2}{2 m_0 r^2}\right] e^{2\gamma r/\alpha m_0} + W_1(\gamma, \alpha, r)$$

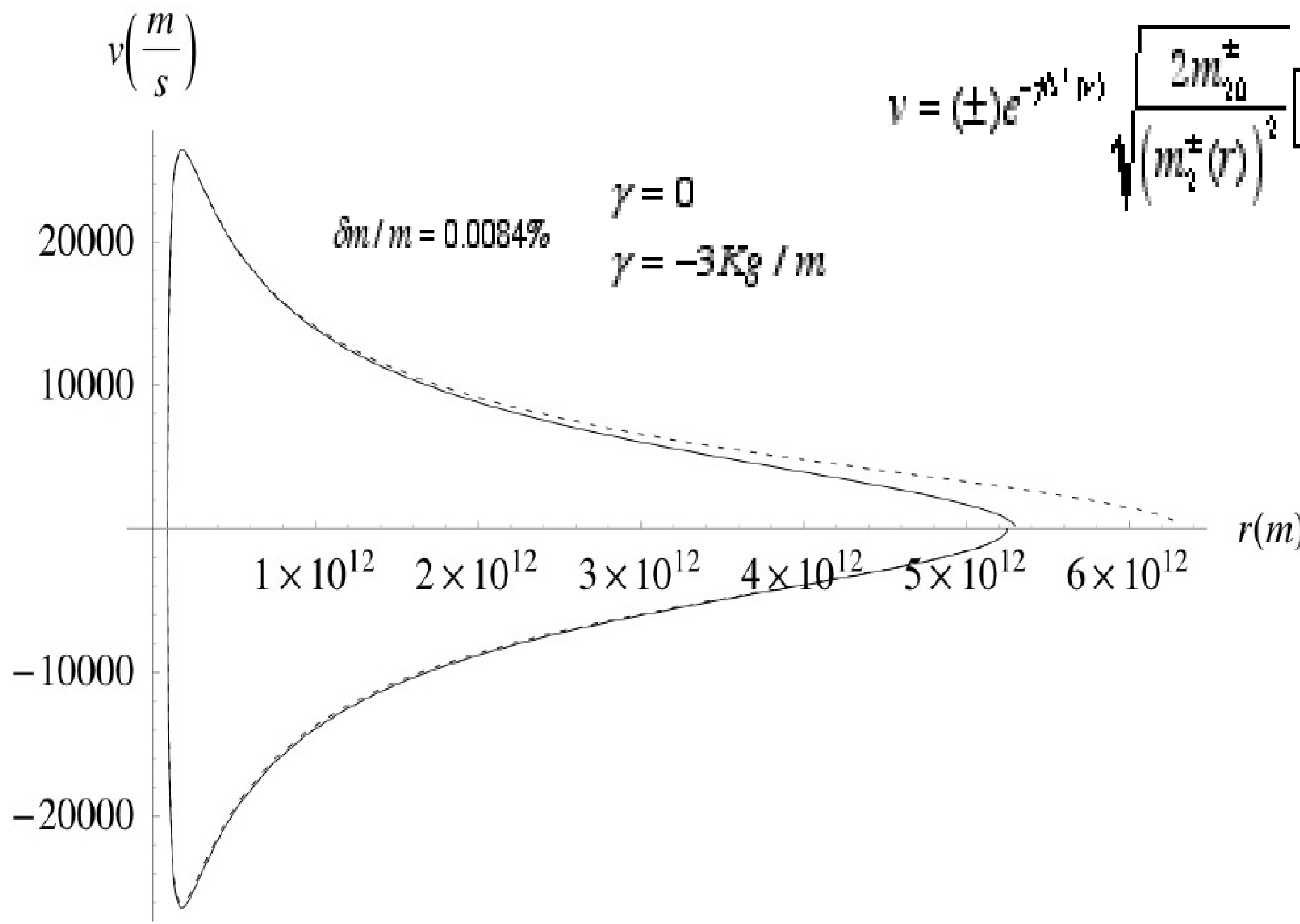
$$V_{eff}^{-}(r) = \left[-\frac{G m_1 m_2}{r} + \frac{l_\theta^2}{2 m_0 r^2}\right] \frac{e^{2\gamma r/(\alpha m_p - b)}}{(m_p - b) e^{2\gamma r \alpha(\alpha m_p - b)}} + W_2(\gamma, \alpha, r)$$


(a)
-V
(1)
(2)
(3)
0
25
50
75
100
125
150
175
200
r
(b)
-P/m0
(1)
(2)
(3)
0
25
50
75
100
125
150
175
200
r

$m_s \approx 1.9891 x 10^{30}\,Kg \quad m_c \approx 2.3 x 10^{14}\,Kg \quad r_p \approx 0.6\,au \quad r_a \approx 35\,au \quad l_0 \approx 10.83\,x 10^{29}\,Kg\,m^2/s$

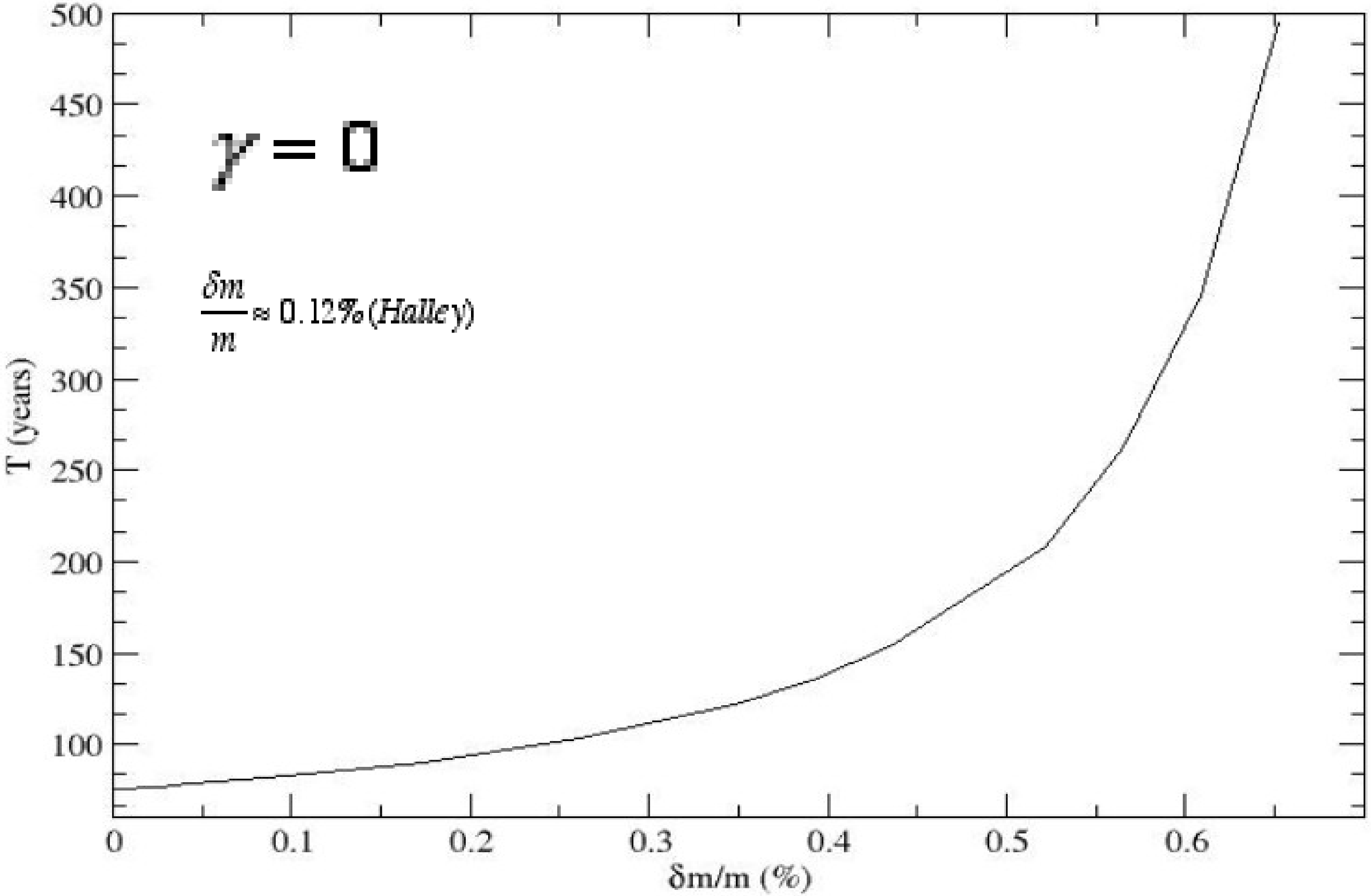
$au = 149597870.7$ km

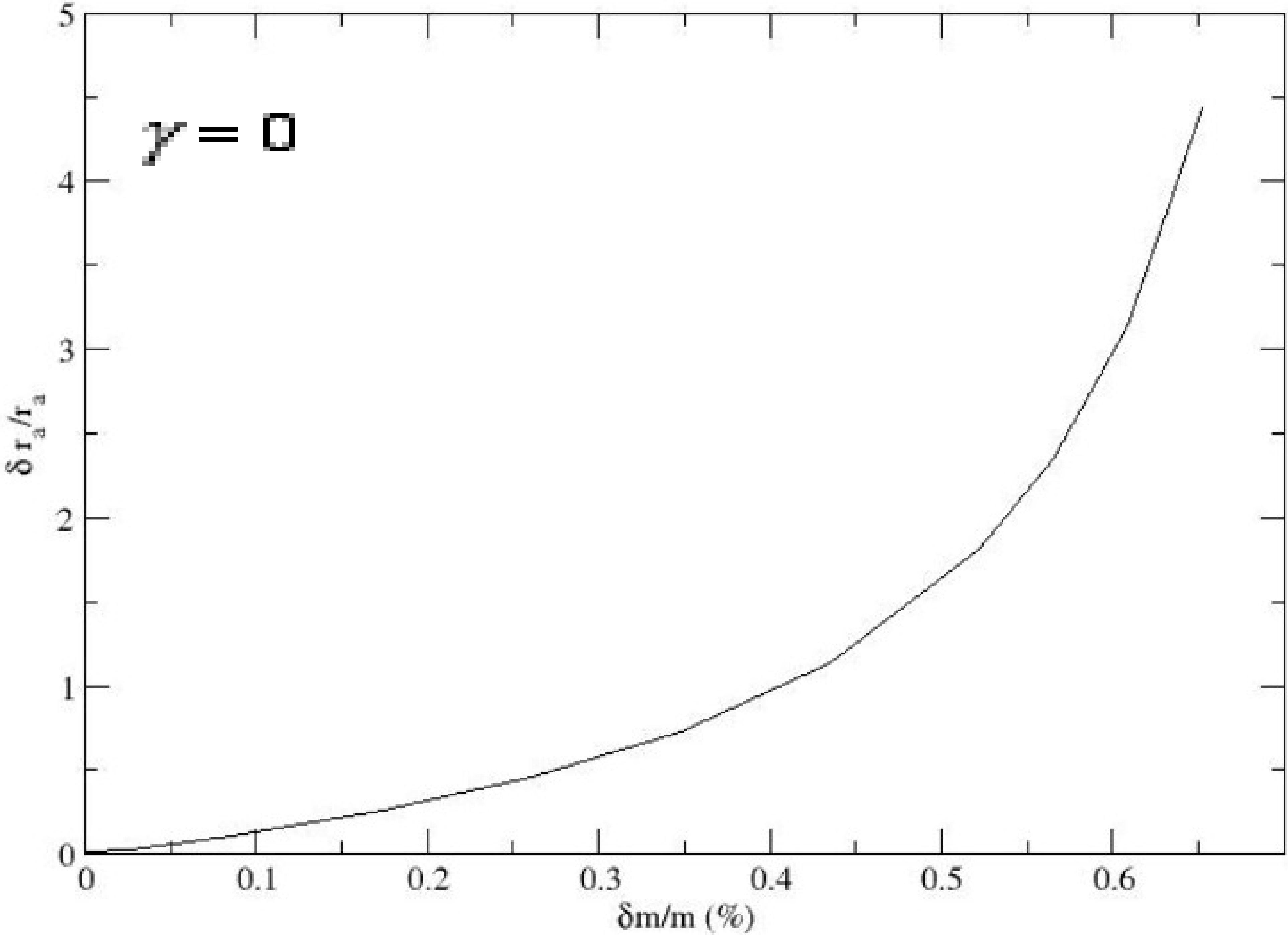
$$v = (\pm) e^{-\gamma s'(v)} \sqrt{\frac{2m_{20}^{\pm}}{\left(m_2^{\pm}(r)\right)^2}} \left[K_0^{\pm} - V_{ef}^{\pm}(r)\right]^{1/2}$$

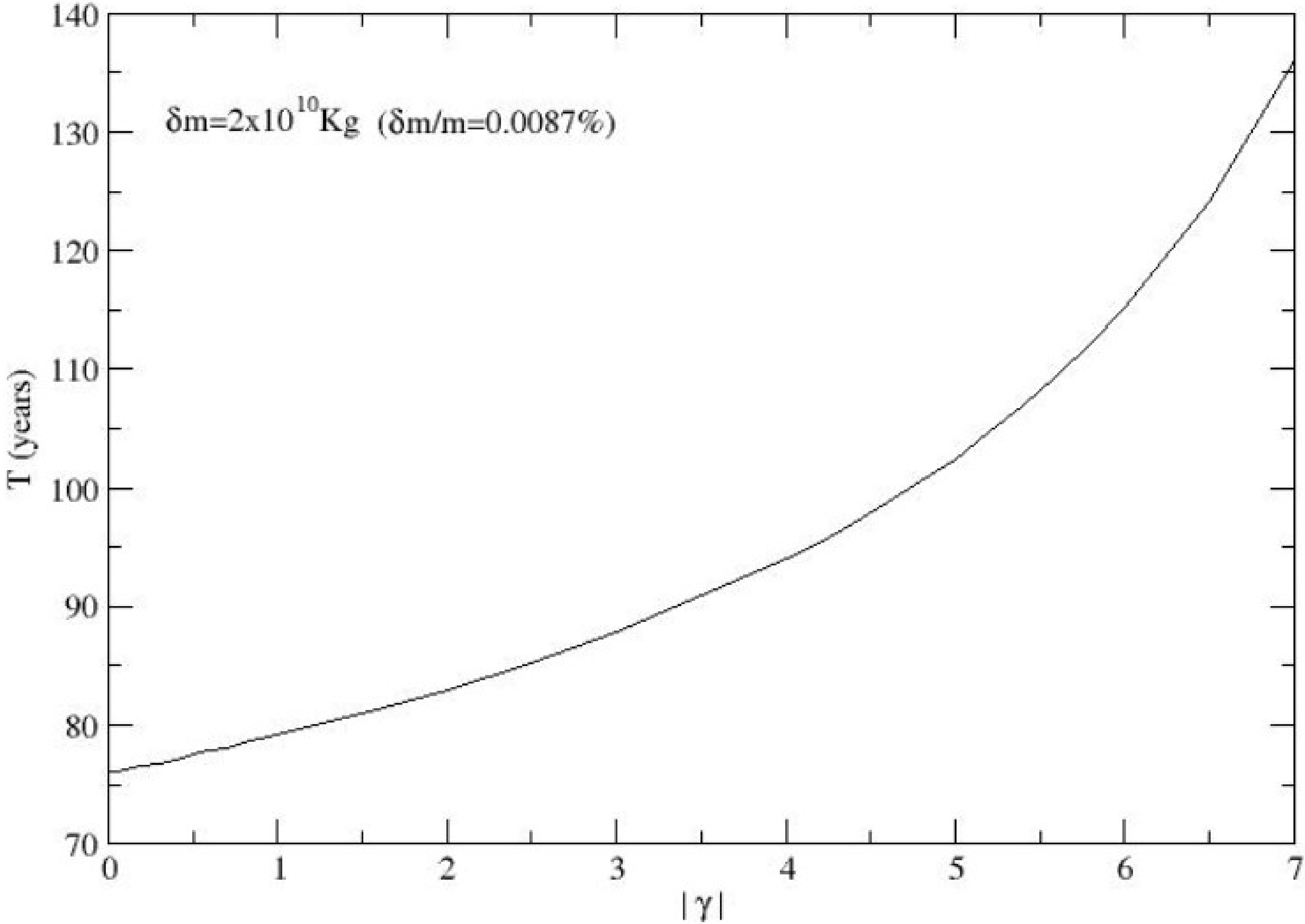


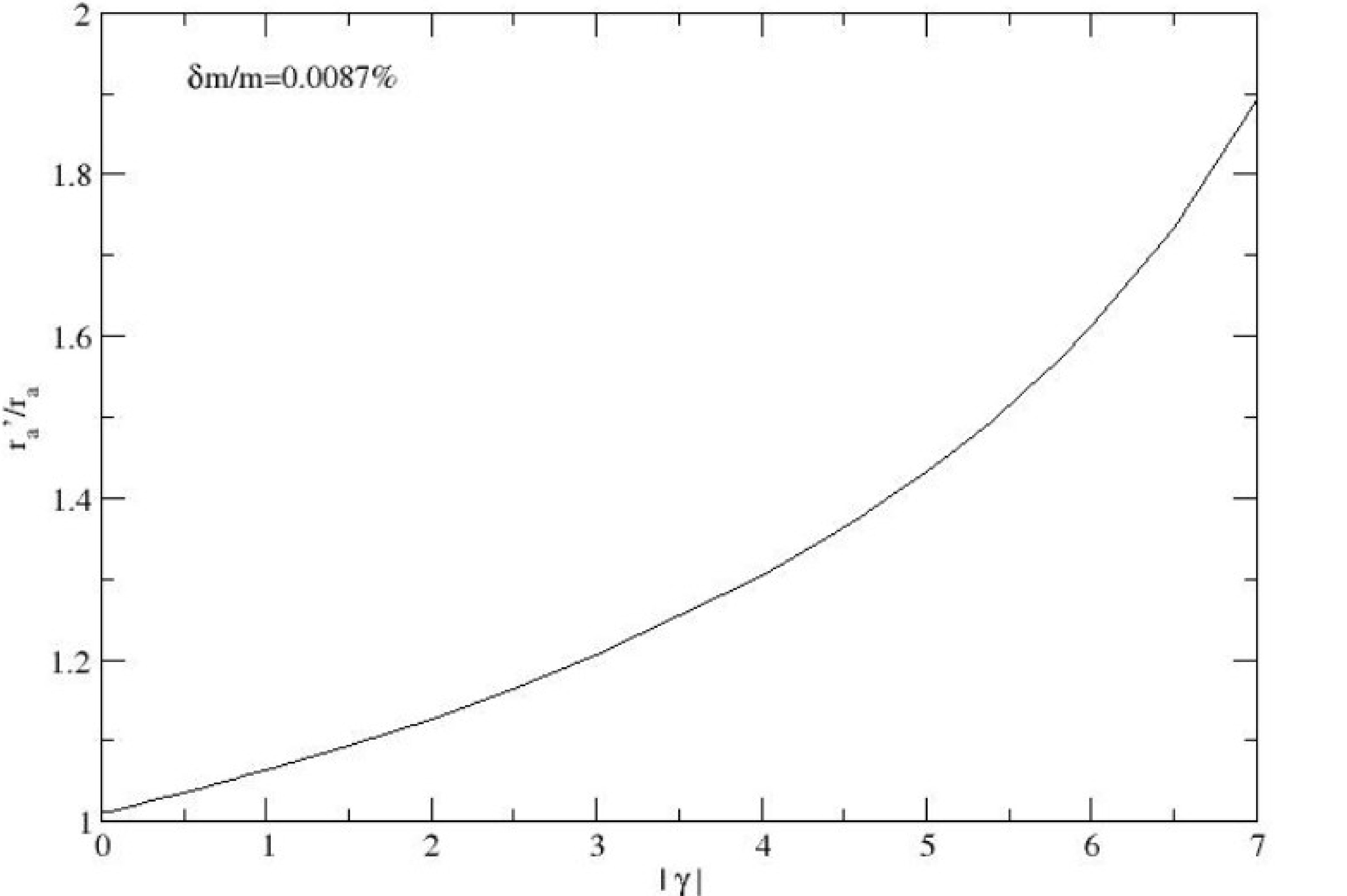
period:

$$T_{1/2}^{\pm} = \frac{1}{\sqrt{2 m_{30}^{\pm}}} \int_{(r_a, r_p)}^{(r_p, r_a)} \frac{m_2^{\pm}(r) e^{\lambda'(r)} dr}{\sqrt{K_0^{\pm} - V_{eff}^{\pm}(r)}} \qquad T = T_{1/2}^{-} + T_{1/2}^{+}$$




γ = 0
δr_a/r_a
δm/m (%)
0
0.1
0.2
0.3
0.4
0.5
0.6
1
2
3
4
5


δm=2x10^10 Kg (δm/m=0.0087%)
T (years)
|γ|
140
130
120
110
100
90
80
70
0
1
2
3
4
5
6
7



δm/m=0.0087%
r_a'/r_a
|γ|
2
1.8
1.6
1.4
1.2
1
0
1
2
3
4
5
6
7

Conclusions:

Taking the reference system in the body of fixed mass, the two 3-D gravitational bodies problem is reduced a 1-D equation. Then, a constant of motion, a Lagrangian and a Hamiltonian are obtained for the radial part of the motion.

A model for the mass variation is given, and the resulting trajectories in the (r,v) space (constant of motion) are different from those in the (r,p) space (Hamiltonian).

Orbits of the comet are not close neither ellipticals but spiral out as the mass of the comet lost . The period of the comet increases with the comet mass lost.

Damping-anti damping force makes the comet to move away even more from the star, increasing the aphelion, perihelion distances and the period of the comet.