# UMBRAL MAPS AND UMBRAL ORTHOGONAL POLYNOMIALS

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The umbral calculus is an old mathematical tool that began its first steps in the XVII century [1]. Since the second half of XIX [2, 3] it was systematically applied although it is in the second half of XX [4, 5] when a formal theory was established.

Recently it has been used to provide discrete representations of canonical commutation relations, like $[x, \partial_x] = 1$, [7]-[12]. This approach can be used to map equations and their solutions from a (continuous) framework to another (discrete) one. This umbral map preserves the point symmetries of the equations but, in general, new symmetries may appear originating a different behavior in some cases [13]-[14].

An umbral version of the orthogonal polynomials is presented. The umbral counterpart of the classical relations, that determine the polyniomials, is obtained [6].

## Referencias

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